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THE SPECTRAL ELEMENT METHOD AS AN EFFECTIVE TOOL FOR SOLVING LARGE SCALE DYNAMIC SOIL-STRUCTURE INTERACTION PROBLEMS

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ABSTRACT

The spectral element method (SEM) is a powerful numerical technique naturally suited for wave propagation and dynamic soil-structure interaction (DSSI) analyses. A class of SEM has been widely used in the seismological field (local or global seismology) thanks to its capability of providing high accuracy and allowing the implementation of optimized parallel algorithms. We illustrate in this contribution how the SEM can be effectively used also for the numerical analysis of DSSI problems, with reference to the 3D seismic response of a railway viaduct in Italy. This numerical analysis includes the combined effect of: a) strong lateral variations of soil properties; b) topographic amplification; c) DSSI; d) spatial variation of earthquake ground motion in the structural response. Some hints on the work in progress to effectively handle nonlinear problems with SEM are also given.

Introduction

Wave propagation phenomena can be studied nowadays thanks to powerful numerical techniques stemming from finite differences (FD, see e.g. Moczo 2003), to finite elements (FE, e.g. Bielak 1999, 2003), boundary elements (BE, e.g. Sanchez-Sesma 1995) and spectral elements (SE, e.g. Faccioli 1997; Komatisch 2003, 2004). Spurred by the computational power made available by parallel computers, these techniques have embraced the area of three-dimensional wave propagation, mostly confined to seismological or geophysical prospecting applications.

In spite of the enormous computational progress in recent years, the application of the previous high performance numerical techniques to 3D problems involving the interaction of structures with the surrounding soil under dynamic loading, such as earthquake, wind, or other vibration sources, has received less attention. These problems are typically defined by three major elements: the source, the propagation path and the structure itself. The accurate modeling of each of such elements is a hard task: the source may not be well defined, such as in the

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earthquake case, or it may involve a very large frequency spectrum, such as for traffic-induced vibrations; the propagation path, in terms of spatial variability of dynamic soil properties, is seldom well constrained by suitable geophysical/geotechnical prospecting; finally, the dynamic behaviour of the structure and the supporting soil may be strongly affected by nonlinear effects, the influence of which can be assessed reliably only in few cases.
The simultaneous presence of all of these elements in the same numerical code makes the problem extremely difficult to be handled from the computational point of view, mainly due to the difference of scale dimensions of the various elements, that may range from fractions of a meter for the structure, up to hundreds or thousands of meters for the propagation path.

In this study a parallel spectral element method for modeling 2D and 3D wave propagation problems and capable of dealing with dynamic soil structure interaction (DSSI) problems is presented. Details of its implementation in a parallel environment is given elsewhere (Maggio 2005). After a short theoretical introduction of the SE approach, we will describe shortly the non linear visco-plastic constitutive model implemented in the numerical code, and will introduce an example of the 3D seismic response analysis of a railway viaduct in Italy.

SEM numerical tool: GeoELSE

The SE approach developed by Faccioli (1997) has been implemented in the computational code GeoELSE (GeoELasticity by Spectral Elements (Maggio 2001), (Stupazzini 2004), (Maggio 2005)), for 2D/3D wave propagation analyses. The most recent version of the code includes: (i) the capability of dealing with fully unstructured computational domains and (ii) the parallel architecture. While the former feature allows to treat problems involving complex geometries, the second is the natural approach for large scale applications.
The spectral element method (SEM) is usually regarded as a generalization of the finite element method (FEM) based on the use of high order piecewise polynomial functions. The crucial aspect of the method is the capability of providing an arbitrary increase in accuracy simply enhancing the algebraic degree of these functions (the spectral degree SD). On practical ground, this operation is completely transparent for the users, who limit themselves to choose the spectral degree at runtime, leaving to the computational code the task of building up suitable quadrature points and new degrees of freedom. Obviously, the increasing spectral degree implies the raise the computational effort of the problem.
On the other hand, one can also play on the grid refinement to improve the accuracy of the numerical solution, thus following the standard finite element approach. Spectral elements are therefore a so-called "h-p" method (Faccioli 1996), where "h" refers to the grid size and "p" to the degree of polynomials.
Referring to Faccioli (1997) for further details, we briefly remind in the sequel the key features of the spectral element method adopted. We start from the wave equation:

$$\rho \frac{\partial \boldsymbol{u}^2}{\partial t^2} = div\sigma_{ij}(\boldsymbol{u}) + \boldsymbol{f}, \quad i, j = 1 \ldots d\,(d = 2,3) \tag{1}$$

where t is the time, $\rho = \rho(\boldsymbol{x})$ the material density, $\boldsymbol{f} = \boldsymbol{f}(\boldsymbol{x},t)$ a known body force distribution and σ_{ij} the stress tensor, and introduce the Hooke's law:

$$\sigma_{ij}(\boldsymbol{u}) = \lambda\, div\boldsymbol{u}\, \delta_{ij} + 2\mu\, \varepsilon_{ij}(\boldsymbol{u}), \tag{2}$$

where

$$\varepsilon_{ij}(\boldsymbol{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{3}$$

is the small-strain tensor, λ and μ are the Lamé elastic coefficient, and δ_{ij} is the Kronecker symbol, i.e., $\delta_{ij} = 1$ if *i=j*, and $\delta_{ij} = 0$, otherwise. As in the FEM approach, the dynamic equilibrium problem for the medium can be stated in the weak, or variational form, through the principle of virtual work (Zienkiewicz 1989), and, through a suitable discretization procedure that depends on the numerical approach adopted, can be written as an ordinary differential equations system with respect to time:

$$[M]\,\ddot{\boldsymbol{U}}(t) + [C]\,\dot{\boldsymbol{U}}(t) + [K]\,\boldsymbol{U}(t) = \boldsymbol{F}(t) + \boldsymbol{T}(t) \tag{4}$$

where matrices $[M]$, $[C]$ and $[K]$, respectively are the mass, the damping and the stiffness matrix, vectors $\boldsymbol{F}$ and $\boldsymbol{T}$ are due to the contributions of external forces and tractions conditions, respectively. In our SE approach, $\boldsymbol{U}$ denotes the displacement vector at the Legendre-Gauss-Lobatto (LGL) nodes, that correspond to the zeroes of the first derivatives of Legendre polynomial of degree N (Abramowitz 1996). The advancement of numerical solution in time is provided by the explicit 2nd order leap-frog scheme (LF2-LF2) (Maggio 1994).
This scheme is conditionally stable and must satisfy the well known Courant-Friedrichs-Levy (CFL) condition:

$$\Delta t \le \gamma \frac{\Delta x_{\min}}{c_{\max}} \tag{5}$$

where $\Delta x_{\min}$ is the shortest spectral grid spacing, $c_{\max}$ is an upper bound for the wave propagation velocity, and γ is a positive constant strictly less than 1.
The key features of the SE discretization are the following:

1. Like in the FEM standard technique, (i) the computational domain may be split into quadrilaterals in 2D or hexahedras in 3D, (ii) both the local distribution of grid points within the single element and the global mesh of all the grid points in the domain must be assigned, (iii) many of these latter are shared amongst several spectral elements, (iv) each spectral element is obtained by a mapping of a *master* element through a suitable transformation and all computations are performed on the master element (Fig. 1). Research is in progress regarding the introduction of triangular spectral elements (Mercerat 2005).
2. The nodes within the element where (i) displacements and spatial derivatives are

computed, (ii) on which volume integrals are evaluated, are not necessarily equally spaced. An example of LGL nodes for spectral elements with different degree is shown in Fig. 2.

3. The interpolation of the solution within the element is done by Lagrange polynomials of suitable degree.

4. The integration in space is done through Legendre-Gauss-Lobatto quadrature formula.

Thanks to this numerical strategy, the exponential accuracy of the method is ensured and the computational effort minimised, since the mass matrix results to be diagonal.

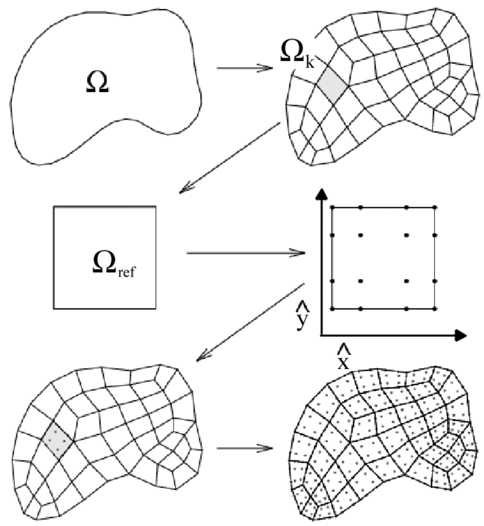


Figure 1. Computational domain is decomposed into a family of non overlapping quadrilaterals, obtained by a mapping of the master element through a suitable transformation (Casadei 2000).

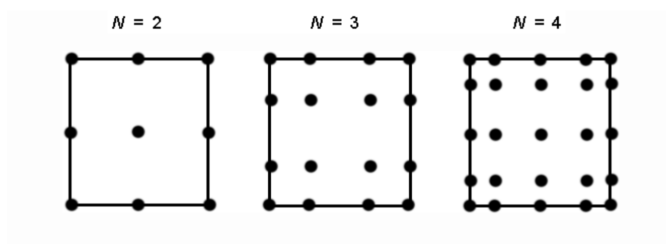


Figure 2. LGL nodes within spectral elements with different degree N.

Constitutive model

The kernel described up to now follows the most simple way of interpreting the dynamic response of soils, that consists in defining an elasto-viscous model. In order to take in account the permanent displacement occurrence and to study the strain localization phenomena, a non-local viscoplastic constitutive model has been implemented in GeoELSE. This choice allows to avoid the loading-unloading criterion definition: it is not necessary to introduce the consistency rule, as the viscoplastic strain rate tensor (Eq. 7) depends directly on the function describing the viscous nucleus and not on the plastic multiplier. For this reason at any time step the irreversible strain rate tensor $\dot{\varepsilon}_{ij}^{vp}$ is known.

The constitutive model implemented in the GeoELSE code is briefly outlined here, with no specific description of the constitutive equations (yield function, plastic potential, hardening rule, etc.) and of the mechanical meaning of the constitutive parameters. For a thorough description of the theoretical aspects reference is made to di Prisco et al. (di Prisco 1996), (di Prisco 2002), (Zambelli 2004). In particular, the non-local viscoplastic version characterised by a set of constitutive parameters depending on the current relative density has been chosen. This is characterised by a single potential, by a non-associated constitutive relationship, by an anisotropic strain hardening and by a visco-plastic Perzyna's type flow rule (Perzyna 1963), (Perzyna 1966).

The model is conceived in small strains; as a consequence, the strain rate tensor $\dot{\varepsilon}_{ij}$ can be defined by the superposition of an instantaneous elastic strain rate tensor $\dot{\varepsilon}_{ij}^{el}$ and a delayed plastic strain rate tensor $\dot{\varepsilon}_{ij}^{vp}$:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{el} + \dot{\varepsilon}_{ij}^{vp} . \tag{6}$$

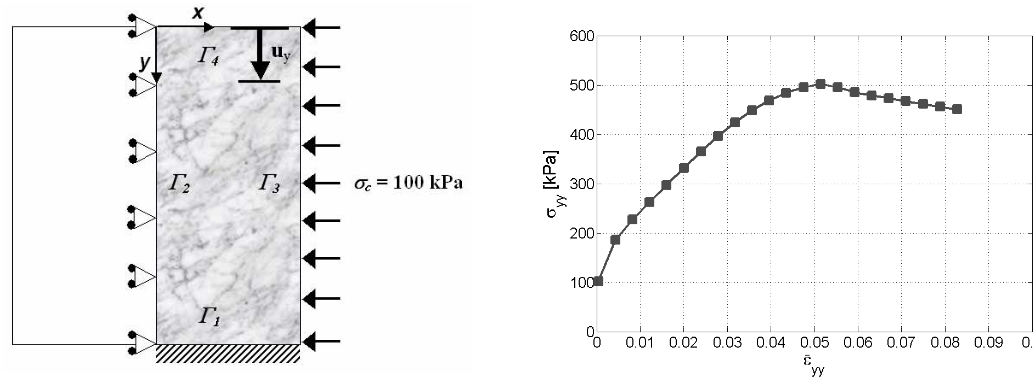
As far as the elastic strain rate tensor is concerned, we must stress that, conversely to what is in the previously cited papers, the compliance matrix is assumed to be isotropic, linear and independent on the effective pressure. The viscoplastic strain rate term is obtained by the Perzyna's approach, i.e.

$$\dot{\varepsilon}_{ij}^{vp} = \phi(f) \frac{\partial g}{\partial \sigma_{ij}^{'}} \tag{7}$$

where *f* is the yield function, *g* the plastic potential, *ϕ(f)* the viscous nucleus and $\sigma_{ij}^{'}$ is the effective stress rate tensor.

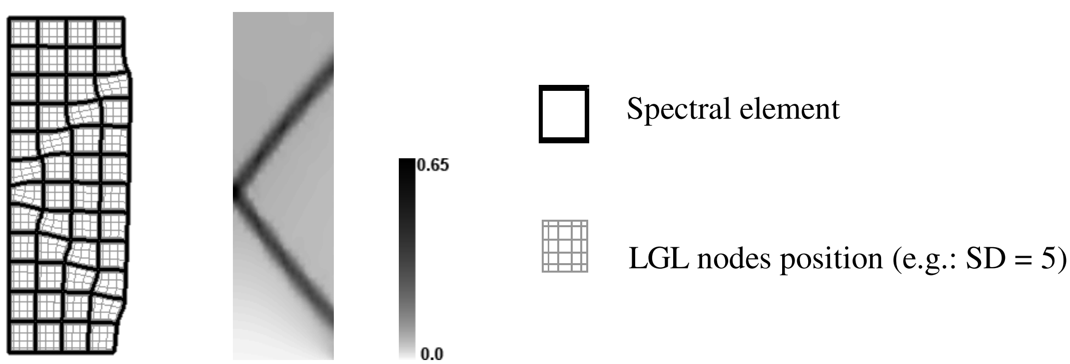
The model is capable of reproducing correctly the mechanical behaviour of granular materials under large strain cyclic loading thanks to the anisotropic hardening, of simulating the static liquefaction of saturated loose sand specimens, and of capturing the softening regime and the progressive decrease in dilatancy of dense sand specimens thanks to the evolution of constitutive parameters on the current relative density.

For example, GeoELSE, equipped with this constitutive model, allows to study the strain localisation in a homogeneous dry dense sand specimen loaded in bi-axial conditions. The sample is first isotropically consolidated under a confining pressure of 100 kPa, then axially loaded (Fig. 3a): the horizontal stress is kept constant while the upper boundary (Γ_4) displacement rate is imposed equal to 1.4 10^{-4} m/s and the lower boundary (Γ_1) is kept fixed. Only one half of the entire specimen is analysed and the left boundary (Γ_2) stands for the symmetry condition (horizontal displacements are not allowed and the surface is assumed to be smooth). The shear band occurrence is evident both from the stress-strain curves (Fig. 3b) and from the snapshot, captured at $\overline{t} = 43\,s$ (Fig. 4).



a) **b)**

Figure 3. Bidimensional dry dense sand specimen (**a**), isotropically consolidated under a confining pressure of 100 kPa, axially loaded with a displacement rate equal to 1.4 10^{-4} m/s, (**b**) stress-strain curves (σ_{yy} stands for the vertical load divided by the initial specimen area, $\overline{\varepsilon}_{yy}$ for the vertical displacement divided by the initial specimen heigth).



a) **b)**

Figure 4. Deformed mesh (**a**) and accumulated irreversible strain tensor modulus (**b**) $\varepsilon_{acc}^{vp} = \sqrt{2/3\ \varepsilon_{ij}^{vp}\varepsilon_{ij}^{vp}}$ relative to the time instant of 43 s obtained by the viscoplastic model with SD = 5.

Applied study case: Acquasanta viaduct (Genova, Italy)

The Acquasanta bridge is a brick masonry railway viaduct consisting of eleven circular arch spans, 18.5 m long, with the four central spans linked by an intermediate deck (Fig. 5, left side). The arches support two side walls, in brick masonry, crowned by a cornice of stone masonry at side-walk level. The maximum height of the walls is 11.8 m and their thickness is 0.9 m. The longitudinal axis of the bridge lies along an arc of circle. The piers are partly founded on weak rock and partly on stiff rock.

The 3D seismic response of the Acquasanta bridge and of the surrounding area due to an incident vertically propagating plane wave is the object of the present study, reported in detail by Stupazzini (2004). Fig. 5 (right side) shows the 3D mesh adopted for the analysis. The investigated area is 2000 m long, 1750 m wide and 860 m deep. The model was designed to propagate up to 6 Hz, with a Ricker type time dependence of the excitation, shown in the bottom part of Fig. 5. The SE mesh consists of 38,569 hexahedral elements with spectral degree 2, corresponding to 323,792 grid points. The hexahedral elements dimensions range from some tens of centimeters to about 1000 m. The memory allocation is around 145 Mb and the single time step is performed in 1.41 s. on a single AMD Athlon XP 2000.
With this numerical mesh, the problem can be handled in its 3D complexity and the following topics have been analysed:
- dynamic soil-structure interaction;
- site amplification effects on seismic wave propagataion due to (i) topographic irregularities, (ii) surface soft soil amplification (caused by the superficial alluvium deposit shown in light blue in Fig. 5), (iii) lateral soil heterogeneity due to a subvertical fault (red line) delimiting schists and serpentine rock.

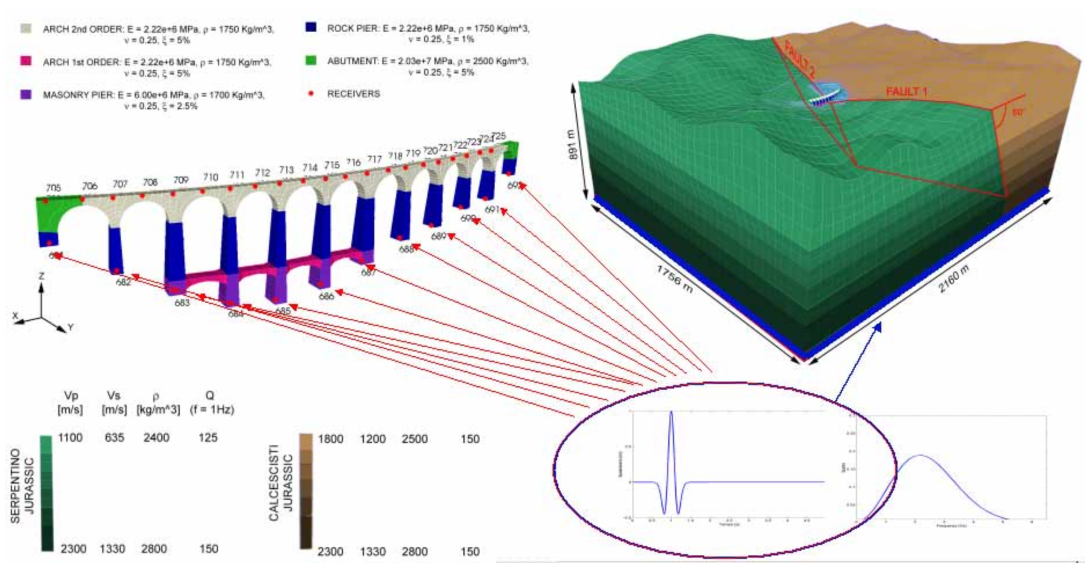


Figure 5. 3D model of Acquasanta bridge and the surrounding geological configuration. The investigated area is 2000 m in length, 1750 m in width and 860 m in depth . The model was designed to propagate waves up to 6 Hz in frequency and has 38569 hexahedral elements and 323792 grid points.

In principle, a near-field seismic source can be also introduced and non linear effects can be handled through the constitutive model introduced previously.
Displacement time-histories were recorded at several receiver points along the bridge, as shown in Fig. 5. Due to the lack of observational data, it was decided to produce a comparison between the results of the mesh previously described, excited by a plane wave Ricker wavelet (f_{max} = 6 Hz, t_0 = 1.2 s) propagating upward, and those obtained from a simplified analysis. The complete model will be denoted as "DSSI model", while the simplified one ("NoSSI model") is obtained

by simultaneously applying to foundation points the same displacement time history, so that soil-structure interaction and wave propagation phenomena in the soil are neglected.
Fig. 6 (left side) shows the comparison between displacement histories of the "NoSSI" and "DSSI" models, recorded from receivers 681 up to 692, located on the base of each pier. It may be noted that:

- topographic effects can be observed comparing record 681 and 685: the former is clearly amplified compared with the latter, which lies on the bottom of a canyon;
- receivers 688, 689 and 690 are amplified because of the alluvium cover;
- different wave propagation velocity in calcareous schists and serpentine rock affects the time of the first wave arrival, even if it cannot be easily recognized.

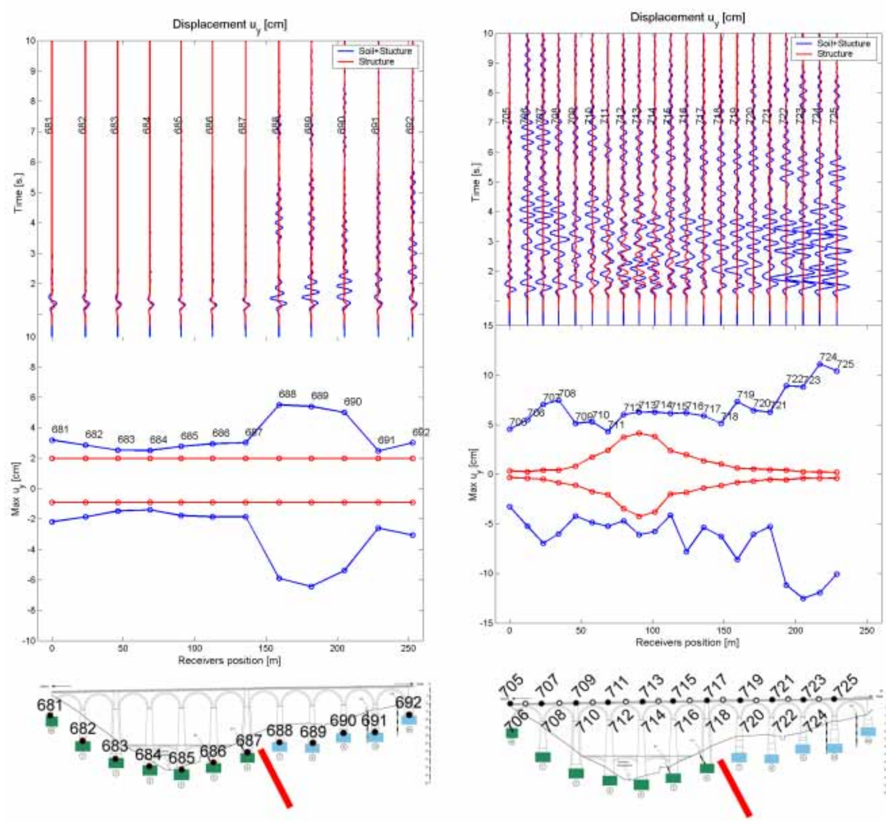


Figure 6 *Left side* - Comparison between u_x of "structure" (red line) and "soil-structure" (blue line) model computed at receivers 681 up to 692, located at the base of each pier. *Rigth side* - Comparison between u_x of "structure" (red line) and "soil-structure" (blue line) model computed at receivers 705 up to 725, located at the top of each pier and in the middle of the main arches.

Fig. 6 (right side) shows the comparison between displacement time-histories of the "NoSSI" and "DSSI" models computed at receivers 705 up to 725, located at the top of each pier and in the middle of the main arches. The differences between the two simulations are significant. Namely, it can be remarked that:

- the maximum values of the DSSI displacements are up to 10 times higher than the NoSSI ones;
- DSSI time histories are significantly more complex compared with the NoSSI ones, along the entire bridge.

CONCLUSIONS

The spectral element method has already become one of the most popular and effective approaches for numerical wave propagation analyses in the seismological field. This paper shows how it can efficiently deal also with challenging engineering problems such as complex 3D dynamic soil-structure interaction analyses, involving strongly heterogeneous media.
Obviously in order to have a tool capable of solving the entire process from the source up to the structure ("end to end simulation") the kernel has been already equipped with an efficient visco-plastic constitutive model. Implementation of the domain reduction method proposed by Bielak (2003), to couple domains of different size, is under way: its effectiveness for SE calculations has already been discussed elsewhere (Faccioli 2005).

Acknowledgments

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