Abstract

In this work, a geometrical model is used to evaluate the sun radiation reflected from the heliostats toward the aim point on the tower top, by taking into account the shading and blocking between neighboring heliostats. This results in an analytical expression for the heliostats efficiency, which can provide very useful informations for the optimization of the heliostat field. In particular, we obtain a very simple and exact expression of the maximal energy collectable by the solar field and present effective strategies to reach such maximum.

Keywords: modeling, shading, blocking

1. Introduction

Solar systems need large dedicated areas. In densely populated regions, where the territory has a high value, it is important to maximize the area exploitation, possibly without using very large amounts of costly reflecting surfaces. Optimization of solar fields is, thus, crucial to the development of solar tower technology. However, in the literature, simple rules or procedures to this scope don't exist, and optimizations are only partially made by using time consuming software tools.

In this work, we overcome this gap. By means of an analytical model, we provide 1) a very simple and exact expression of the maximal energy collectable by the solar field and 2) a simple and effective strategy to reach such maximum.

2. The model

Fig. 1. Geometry of the tower system
In Fig. 1, the geometry of the sun-heliostat-tower system is schematized, and the relevant geometrical quantities are defined: the tower focus is the aim point toward whom the sun rays are reflected, O and P are the projections of the focus on the ground plane along the directions of its normal and of the sun rays, respectively. \( \theta_s \) and \( \theta_t \) are the angles formed by the incident and reflected ray to the considered heliostat with the ground normal, and \( \theta_s \) and \( \theta_t \) are their projections on the sun-heliostat-focus (SHF) plane; \( \beta \) is the inclination of the SHF plane from the normal to the ground. The angles \( \theta_s \), \( \theta_t \), \( \theta_s \), \( \theta_t \) and \( \beta \) are related by:

\[
\cos(\theta_s) = \cos(\theta_t) / \cos(\beta) \\
\cos(\theta_t) = \cos(\theta_t) / \cos(\beta)
\]

(1)

The angle \( \alpha \) of incidence and reflection on each heliostat is given by \( \alpha = (\theta_s + \theta_t) / 2 \).

The simpler way to evaluate the effects of shading and blocking, is by first considering the shadows in the SHF plane (longitudinal direction in Fig. 2) and, afterwards, by evaluating the possible further shading along the normal direction (dashed line in Fig. 2). We define \( d \) as the distance of neighboring mirrors and \( l \) as the mirror length.

![Fig. 2. Scheme of the solar tower system](image)

In Fig. 3, the mirrors are represented by the thick lines. \( d^* \) is the distance of the neighboring mirrors in the SHF plane and \( l^* \) is their length. \( l^*_{\text{eff}} \), represents the effective length of sun collection of the considered heliostat in the longitudinal direction. It is seen that, when the images of the neighboring mirror are not overlapping (no blocking nor shading – Fig. 3, left), \( l^*_{\text{eff}} \) is given by:

\[
l^*_{\text{eff}} = l^* \cos(\alpha) \quad \text{(2)}
\]

On the other hand, when the images overlap (shading – Fig. 3, right), we have:

\[
l^*_{\text{eff}} = d^* \cos(\theta_s) \quad \text{(3)}
\]

It is easy to see that, when \( \theta_t \) is larger than \( \theta_s \) (blocking), the same relations hold with \( \theta_t \) in spite of \( \theta_s \):

\[
l^*_{\text{eff}} = d^* \cos(\theta_t) \quad \text{(4)}
\]

Overall, we have:

\[
l^*_{\text{eff}} = \min\left(d^* \cos(\theta_s), d^* \cos(\theta_t), l^* \cos(\alpha)\right) \quad \text{(5)}
\]
When considering the shading in the normal direction (dashed line in Fig. 2) the situation is even simpler, since, as shown in Fig. 4, the projections of sun and focus directions coincide and are orthogonal to the heliostats. Therefore, we get:

\[ l_{\text{eff}}^T = \min \left| d^T \cos(\beta), l^T \right| \]  

Overall, the energy that the mirror \( m \) reflects toward the tower, is simply equal to the direct normal irradiation \( G_b \) multiplied by the effective heliostat area \( A_{\text{eff}} \), which is the product of \( l_{\text{eff}}^T \) times \( l_{\text{eff}} \):

\[ E = G_b A_{\text{eff}} = G_b l_{\text{eff}}^T l_{\text{eff}} \]  

By dividing the energy by the territory surface area \( S \) and by \( G_b \), we get the collection efficiency \( \eta \), which is a measure of the territory exploitation:

\[ \eta = \frac{E}{SG_b} = \frac{l_{\text{eff}}^T l_{\text{eff}}}{S} \]  

By considering that:

\[ \frac{d}{S} = \frac{d^T}{1} = \frac{l_{\text{eff}}^T l_{\text{eff}}}{K} \]  

where \( K \) defines the coverage, i.e. the ratio between heliostat surface and territory area. By using Eq. 1-6 we get:

\[ \eta = \min \left( \cos(\theta_s), \cos(\theta_t), K \cos(\alpha), \frac{l_{\text{eff}}^T}{d^T} \cos(\theta_s), \frac{l_{\text{eff}}^T}{d^T} \cos(\theta_t), \frac{l_{\text{eff}}^T}{d} \cos(\alpha) \cos(\beta) \right) \]  

In the following, we show how to eliminate some of the terms in the relation above, with the result of having a higher efficiency and a simpler relation. In section 2.1, we will study the case of uniform coverage, where
K is a fixed constant; afterwards, in section 2.2, such condition is relaxed and the general case is considered.

2.1 Uniform coverage

The last three terms in Eq. 10 correspond to heliostats which “overlap” (shading or blocking) in one direction while they do not overlap in the other direction. By keeping the heliostat position and area $A$ constant, it is always possible to modify the heliostat shape in such a way that either the heliostats overlaps in both directions or in none. This can be done, for example, by imposing:

$$l_{\text{opt}}^n = d_{\text{min}} \cos(\theta_s), \cos(\theta_t), K \cos(\alpha),$$ (11)

and

$$l_{\text{opt}} I_{\text{opt}}^T = A.$$ (12)

By doing this, we can neglect the last three terms and Eq. 10 becomes:

$$\eta_{\text{opt}} = \min \cos(\theta_s), \cos(\theta_t), K \cos(\alpha).$$ (13)

This simple relation for the collection efficiency, which can be obtained by the procedure above, represents a superior limit for any real heliostat field and can be easily used to assess its quality:

$$\eta \leq \min \cos(\theta_s), \cos(\theta_t), K \cos(\alpha).$$ (14)

An example of this use is given in the results section.

Even if derivation of Eq. 14 in this paper has been not completely trivial, its physical interpretation is quite intuitive: the first term represents the sun radiation reaching the ground surface (the collected radiation can’t be larger than the one reaching the surface of the heliostats field); the second term represents the fraction of surface “seen” from the tower focus (the collected radiation can’t be larger than the one reaching the focus); the third term, represents the well known “cosine effect” (the collected radiation can’t be larger than the total radiation reflected by the heliostats). In the limit of low coverage, the last term dominates, while for high coverage the first two prevail.

2.2 General case

If we can impose, either by changing the heliostat shape $(l)$ or distance $(d)$ in the two directions, that

$$l^n \cos(\alpha) = \min (d^n \cos(\theta_s), d^n \cos(\theta_t)),$$ (15)

and that

$$I^T = d^T \cos(\beta),$$ (16)

we can exclude shading and blocking between heliostats. Then, from Eq. 5, 6, 8 and 9, we get:

$$\eta = \min \cos(\theta_s), \cos(\theta_t).$$ (17)

The conditions above correspond to a coverage changing as:

$$K_{\text{opt}} = \min \cos(\theta_s), \cos(\theta_t) / \cos(\alpha)$$ (18)

The conditions Eq. 15 and 16, can be used to build high coverage heliostat fields with minimal shading and blocking. An example of this use is given in the results section.

3. Results and discussion

The equations derived in the previous section aren’t based on approximations, and, therefore, are exact.
However, in order to substantiate such a statement, and to demonstrate some possible applications of the analytical model, we performed a series of numerical calculations on solar tower field by using the CRS4-2 code developed within our group [1,2].

As a first application, we use Eq. 14 to assess the quality of a series of heliostat fields. The simulations are performed with fields of circular heliostats positioned on a regular triangular grid. The solar field has the shape of a circular crown with a minimum radius $R_{\text{min}} = 30$ m and a maximum radius $R_{\text{max}} = 140$ m; the relevant field parameters are summarized in Table I.

<table>
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<th>N. heliostats</th>
<th>2716</th>
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<tbody>
<tr>
<td>Land coverage, %</td>
<td>25, 45, 91</td>
</tr>
<tr>
<td>$R_{\text{min}}$, m</td>
<td>30</td>
</tr>
<tr>
<td>$R_{\text{max}}$, m</td>
<td>140</td>
</tr>
<tr>
<td>Tower height, m</td>
<td>50</td>
</tr>
<tr>
<td>Heliostat shape</td>
<td>Circular and rectangular</td>
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<tr>
<td>Heliostat radius, m</td>
<td>1.31, 1.76, 2.5</td>
</tr>
</tbody>
</table>

Table I. Heliostat field parameters

The three different values of land coverage $K$ are obtained by changing the heliostats radius.

In Fig. 5, we compare the numerical energy with the theoretical limit provided by Eq. 14, for three different levels of ground coverage and three different sun elevations (zenit: 0°, 45°, 80°).

It is seen that in both the high and low coverage limit, the numerical energy is very close to the theoretical limit, while in the intermediate case ($K=0.45$), with the sun zenith at 0° (vertical) and 45°, the “real” field energy is about 10% below the limit, and an optimization appears very useful.

In order to illustrate the possible use of Eq. 11 and 12 for optimization purposes, we have examined the energy collected by each heliostat in these two cases. For the sake of simplicity, in this example we have considered an original field of square heliostats, and, by keeping the heliostats on the regular triangular grid, we have used Eq. 11 and 12 to modify the shape of the heliostats, which are becoming rectangles of the same area (in principle, the same result can be reached by keeping the squared shape of the heliostats and modifying the field geometry). In Fig. 6, the collection efficiency $\eta$ is shown for each heliostat. The red line represents the theoretical limit given by Eq. 13, the black dotted line the efficiency of the original field of square heliostats and the blue dashed line the efficiency of the rectangular heliostats modified according to Eq. 11 and 12.
By comparing the red and the dotted curve in Fig. 6, we observe that only a fraction of the squared heliostats reaches the efficiency limit, while most of them are under-performing. On the other hand, the efficiency of the optimized heliostat field (blue dashed lines) is extremely close to the theoretical limit.

Finally, we have checked the possibility to build a high coverage heliostat field with no blocking or shading, by using Eq. 15 and 16. In this example, the sun zenith is at 45°. The heliostats are circular with a 2.5m radius, and their positions are fixed by keeping the distances between neighboring heliostats according Eq. 15 and 16. The resulting coverage is of 62%. We have used the numerical code CRS4-2 in order to check the possible presence of blocking or shading. The relevant parameters are summarized in Table II and the results are presented in Fig. 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<td>N. heliostats</td>
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<tr>
<td>Land coverage, %</td>
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</tr>
<tr>
<td>Rmin, m</td>
<td>30</td>
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<tr>
<td>Rmax, m</td>
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<tr>
<td>Tower height, m</td>
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<td>Heliostat shape</td>
<td>Circular</td>
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<tr>
<td>Heliostat radius, m</td>
<td>2.5</td>
</tr>
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</table>

Table II. Heliostat field parameters

We observe that, because of the approximations used in building the field, some shading and blocking is still present, but its amount is extremely small and negligible.
4. Conclusions

- Optimization of heliostat fields is necessary to avoid territory and heliostats waste
- We provide a very simple limiting expression which can be used to assess the quality of heliostat fields
- We provide effective strategies to reach the maximal theoretical efficiency
- By using the present model it is possible to build high coverage fields with negligible shading and blocking
- This model is an excellent starting point to build optimization procedures taking into account territory and heliostats cost and yearly sun position distributions. Work is in progress on this point.

Because of its simplicity and effectiveness, we believe that this approach can allow a real breakthrough on the design of solar tower heliostat fields.

Acknowledgements

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References