Percolation Transition in Simulated Urban Traffic

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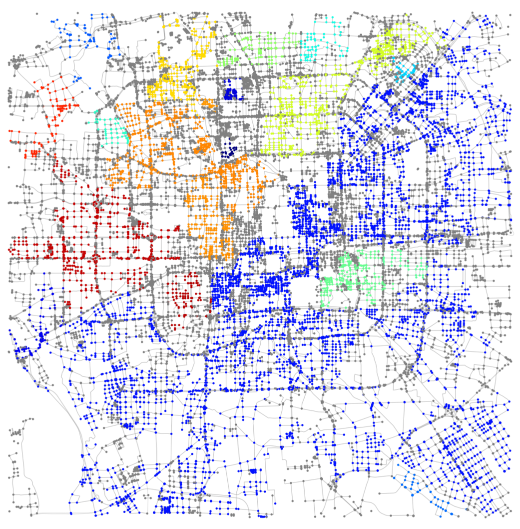
Percolation in Urban Networks



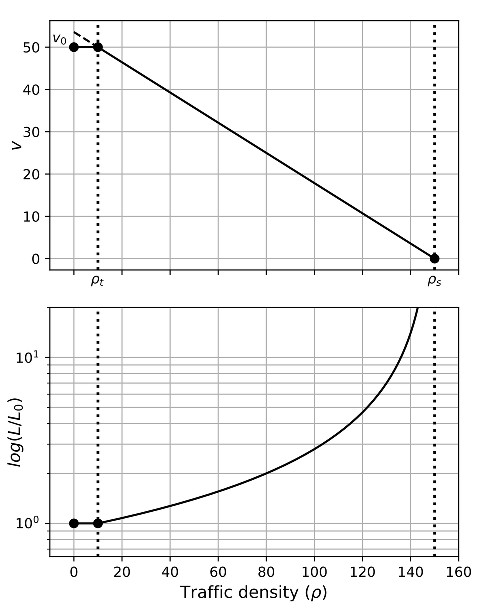
Beijing fragmented network near criticality.

Urban traffic is known to undergo a phase transition when load goes over some threshold[1]. A different kind of critical behavior is embodied by the percolation transition, recently demonstrated for real city traffic[2], in which the effective topology of the network is progressively fragmented by the emergence of bottleneck roads, unable to sustain traffic above some threshold speed. At criticality, the size distribution of the network's connected components follows a power law with a specific critical exponent that depends on the traffic regime[3].

Here we present a set of results produced by: **1)** a simple traffic generation model that is able to mimic some important properties of the real phenomenon; **2)** random edge percolation data (with and without spatial correlation) for real cities (OpenStreetMap); **3)** UBER data under different traffic scenarios for a few cities.

We identify the velocity critical threshold by studying the second largest subgraph of the cluster set: as soon as it reaches the maximum size, that will be the critical velocity and the cluster sizes will have a power-law distribution. In particular, we focus on how the percolation critical exponent τ depends on traffic intensity. This exponent τ governs the behavior of the size scaling $n_s \sim s^{-\tau}$ of functional clusters of the traffic network.

Vehicle Interaction Model



Model speed (top) and effective length (bottom) vs Density.

Our model is able to generate synthetic urban traffic over real, large-scale, full resolution (OpenStreetMap) networks. It is vaguely related to Echenique et al.[4], but introduces the idea of vehicle probability density in place of explicit space-time trajectories and node queues. The principal difference is that real edge attributes such as length, direction, speed limit and the number of lanes are taken into account with respect to a square lattice of the original model. Some cities lack speed limits data for a relevant fraction of edges. Some model properties: **1)** Fundamental Traffic Diagram obtained from real urban data ($\sim$ 200 real time sensors in Cagliari and Torino, Italy) **2)** Constant speed, variable effective road lengths $\rightarrow$ variable path lifetime **3)** Each vehicle is represented by a probability density along a path: Summing it over all vehicles one obtains the total number of equivalent vehicles at all times for each graph edge. We assume that path origins and destinations to be proportional to graph node geographic density: this approx. may be not sufficient for non homogeneously inhabited areas such as skyscrapers near residential single-story buildings, etc. In our model target velocity is the same for all vehicles and constant in time, so, in order to describe slowing down, each edge length grows with local density:

$$\begin{cases} L_i = L_i^0 \frac{(\rho^s - \rho^t)}{(\rho^s - \rho)} & \rho^t \leq \rho < \rho^s \\ L_i = L_i^0 & \rho < \rho^t \\ L_i = \infty & \rho \geq \rho^s \end{cases}$$

We assume that the spatial probability distribution for the k-th vehicle is uniform along its path P_k. The equivalent number of vehicles N_i^k over the edge E_i due to path P_k, is proportional to the geographical length L_i of the edge itself and inversely proportional to P_k total geographical length $l(P_k)$: $N_i^k = \frac{L_i^0}{\sum_{j=1}^N 1_{E_j \in P_k} L_j^0}$, thus the total equivalent number of vehicles N_i over the i-th edge, is obtained by summing contributions due to all paths (vehicles) using E_i: $N_i = \sum_{k=1}^M 1_{E_i \in P_k} N_i^k$, so the density on that edge is simply: $\rho_i = \frac{N_i}{L_i^0} = \sum_{k=1}^M \frac{1_{E_i \in P_k}}{\sum_{j=1}^N 1_{E_j \in P_k} L_j^0}$.

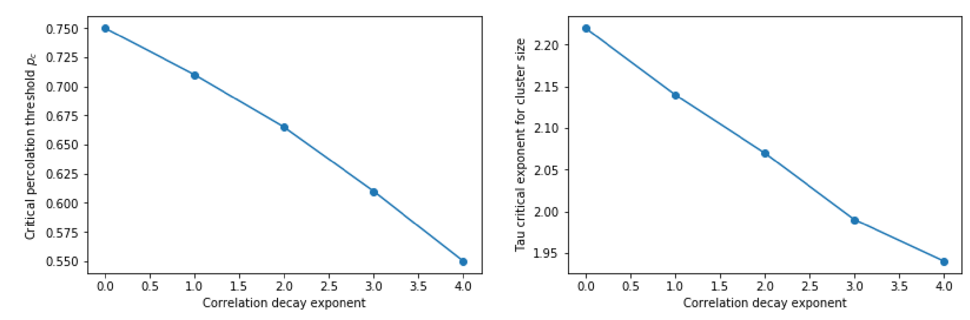
Vehicle Route Generation / Evaporation

Visual correlation between node density and population for Torino.

The network starts as an empty graph with no vehicles. We set a target traffic load for the simulation (number of coexisting vehicles on average) and add a path at each timestep if the load is under the target. Thus traffic is represented by a dynamic set of paths, one for each vehicle, starting from and terminating in uniformly random nodes. A new vehicle is created by associating it to the shortest path (weighted or not) on a dynamic topology graph, whose edges are switched on or off depending on the local vehicle density with respect to a threshold. At each timestep we check which vehicles did reach their destination and their paths are entirely removed from the network. Removing the old path means that local vehicle density is recomputed on all affected edges and re-checked against the threshold to decide if they have to be switched back on. A vehicle reaches its destination after a time proportional to its effective length, equal to the geodetic distance multiplied by a slowdown factor derived from the fundamental traffic diagram[1] for typical urban mobility.

Results

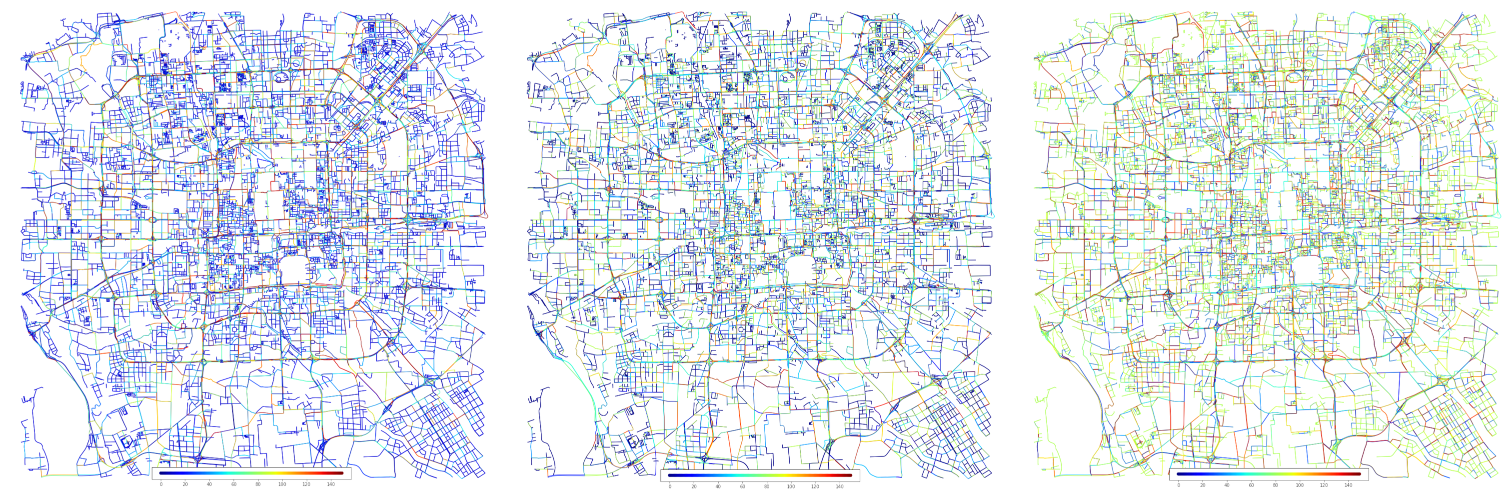
Random percolation with and without correlation - For each urban network we identify the percolation transition threshold (Beijing $p_c \sim 0.75$, London $p_c \sim 0.81$) for uncorrelated noise. We need to compute this for several cities to obtain a p_c distribution. With no correlation the τ critical exponent is between the theoretical results for mean field networks ($\tau = 2.4$) and a square lattice $\tau = 2.05$. Increasing correlation, both p_c and τ decrease in a quasi linear fashion. When comparing random uncorrelated percolation on the urban network graph (Beijing, 14km radius) to GPS data[3], the results qualitatively agree with off-peak traffic, while long range correlated noise induces a τ value similar to rush-hour conditions.



Percolation threshold p_c (left) and critical exponent τ (right) dependence on the spatial correlation length.

Variable spatial correlation on the weighted directed network is obtained by using a Graph Fourier Transform (PyGSP[5]) with different filter power-law decays.

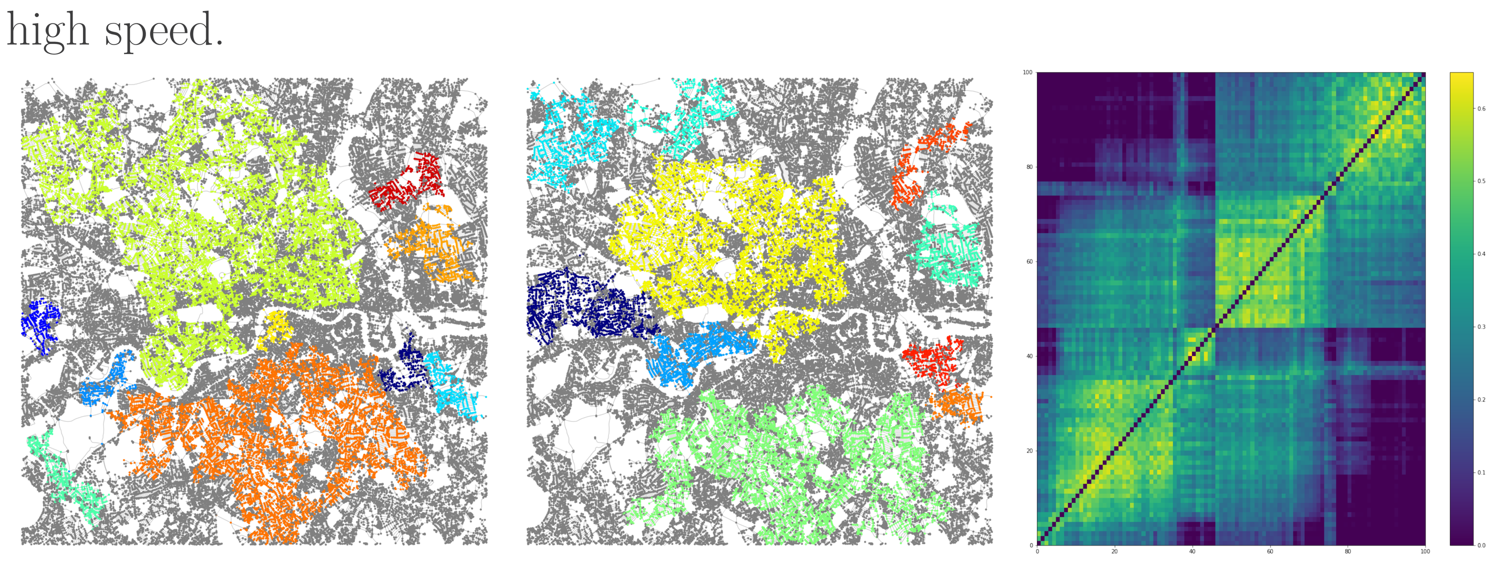
Simulated traffic structure and percolation for Beijing - The simulated traffic for Beijing (R=14km) and London (R=14km) produced critical exponents that slightly differ between the low-traffic and the high-traffic scenarios: the τ value goes from ~ 2.1 to ~ 2.0 respectively. This discrepancy with Ref.[3] for GPS data in Beijing may be due to the absence of information about most speed limits in the OSM map (Beijing) and to model deficiencies such as naive route selection. These two factors could lead to percolation exponent values lower than in the mean field case $\tau \sim 2.4$ since urban highways are underused and don't act as efficient network shortcuts as they should. Nonetheless, at low traffic, highways display very high vehicle densities while the whole network is almost free.



Vehicle density per edge for low, average and high traffic scenarios in Beijing.

Real traffic percolation - UBER data for London (R=14km) produced critical exponent values of about $\tau \sim 2.1$ for off-peak periods and $\tau \sim 2.0$ for rush hours when considering 3 months of data. We plan to repeat this analysis for a few other cities that offer UBER speed data. This result agrees with our simulator for the city of London. London is one of the few cities for which the UBER velocity data is large enough to be used for a percolation analysis.

Stability of the main functional traffic cluster - We finally study the stability of the largest functional cluster over several realizations of the simulated traffic by computing the overlap between replicas and we observe that a complex hierarchy exists. A qualitatively similar result exists for the second and smaller clusters. This result implies that clusters may have an intrinsic stability and their shape is not totally random. This is interesting since we can explore all possible traffic breakup patterns and their relative probability. Moreover, we could think of practical ways of merging clusters to maintain a larger part of the city connected at high speed.



Two realizations of largest critical clusters for London and the overlap matrix for 100 traffic replicas.

References

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