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acoustic microscopy, it is not uncommon to encounter percent bandwidths which exceed 80% [6], [22]. Indeed, ultrasound pulses shorter than a few cycles of the nominal frequency are commonly used to enhance the axial resolution. Since an increase of the bandwidth smears out the side-lobe peaks, the narrow-band analysis can be considered as a worst-case operation and gives only a rough estimation of the performances of a wide-band ultrasound system [23]. In an optimization process, where the challenge is the best achievable performance, the narrow-band array synthesis is not satisfactory.

In this paper, we apply a stochastic optimization technique to a linear array for the wide-band excitation case. The beam pattern formulation of a wide-band linear array with N omnidirectional elements is presented in Sec. II. The evaluation of the beam pattern constitutes the major computational load in the optimization process. In order to reduce the computational burden, a fast, but accurate, numerical routine has been implemented. Within stochastic optimization procedures, the simulated annealing algorithm is an e cient strategy for solving optimization problems with multiple extrema, because its random-search character helps to escape from local minima [24]-[26]. Simulated annealing is an iterative algorithm which starts with an initial random con guration of state variables, and at each step slightly perturbs the con guration. The effects of the perturbation are evaluated according to the values of an energy function E: if the energy is lowered, the perturbation is accepted; if not, the perturbation is accepted with a probability depending on a control parameter T, called temperature. The temperature indicates the probability that a worse con guration is accepted, and is gradually lowered during the running time. Simulated annealing, and other similar algorithms based on combinatorial statistics, such as Monte Carlo methods and genetic algorithms, have been successfully adopted in the sparse array optimization in the narrow-band case [13]-[18], [27]. We use an optimization algorithm, very close to simulated annealing, named threshold accepting [28]-[30]. The basic difference between simulated annealing and threshold accepting is the acceptance rule. It has been shown [28] that threshold accepting yields better results than simulated annealing, even in a shorter computational time. The details of the optimization routine are described in Sec. III.

In Sec. IV, we address the problem of tapering the aperture of a wide-band dense linear array, applying the proposed optimization process. Aperture tapering lowers the side-lobe level and increases the main-lobe width, enhancing the contrast resolution, at cost of a loss of detail resolution [31], [32]. A large number of window functions have been studied in literature and their influence on the beam pattern radiated by a narrow-band array is easily predicted [31]-[35]. However, the beam patterns of wide-band and narrow-band arrays are comparable only in a region close to the focusing direction, where all the transmitted pulses interfere; outside from this region the pressure energy radiated by a wide-band array smears out in time [6]. We show that a wide-band optimization process yields better results than classical shading functions.

In Sec. V, we consider the optimization of the layout (positions and weights) of a linear sparse array in order to achieve a desired beam pattern with a xed, or a minimum, number of array elements. In

particular, we compare the results obtained by narrow-band and wide-band optimization, showing that better results can be achieved with the proposed approach.

In order to give a further comparison with alternative methods for array synthesis, the beam pattern achieved with the proposed method is contrasted with those generated by a wide-band sparse random array and a wide-band 2λ-spaced periodic array.

Finally, some hints for future developments are given in the conclusive section.

II. Beam pattern of a wide-band linear array

We consider a linear array constituted by N omnidirectional sources uniformly spaced along the $x-$axis, which radiate into a homogeneous, non dispersive and lossless medium. The acoustic pressure field in the far-field region is calculated as a function of the observation angle θ and the steering angle θ_0, which are measured with respect to the $z-$axis, perpendicular to the array, as shown in Fig. 1

$$p(\theta, \theta_0, t) = g(t) * h(\theta, \theta_0, t). \tag{1}$$

Here the asterisk denotes convolution and $g(t)$ is the ultrasound pressure pulse transmitted by a single element with a carrier frequency f_0. The array impulse response $h(\theta, \theta_0, t)$ can be expressed as

$$h(\theta, \theta_0, t) = \sum_{n=0}^{N-1} w_n \delta\left[t - \frac{nd(\sin\theta - \sin\theta_0)}{c}\right], \tag{2}$$

where, $\delta(\cdot)$ is the Delta function [36], w_n the amplitude weight of the $n-$th element, c the sound speed in the medium and d the array element spacing. The spacing is assumed $d = \lambda/2$ ($\lambda = c/f_0$). To reduce the number of parameters, we consider the array impulse response as a function of the real valued variable $u = \sin\theta - \sin\theta_0$, which ranges between -2 and 2. Then, Eq. (2) becomes

$$h(u, t) = \sum_{n=0}^{N-1} w_n \delta\left(t - \frac{nu}{2f_0}\right). \tag{3}$$

We assume that the ultrasound pulse transmitted by a single array element $g(t)$ has a Gaussian frequency spectrum with a -6 dB relative bandwidth B

$$g(t) = u_0 \exp\left[-\frac{\pi^2 B^2 f_0^2 (t-\tau)^2}{\alpha}\right] \sin(2\pi f_0 t)\,\mathrm{step}(t), \tag{4}$$

where u_0 is a constant, $\tau = 1.5/Bf_0$, $\alpha = 1.2\ln 10$, and $\mathrm{step}(t)$ is the unity step function

$$\mathrm{step}(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{elsewhere.} \end{cases} \tag{5}$$

In order to remove the dependence on the frequency f_0, we rewrite Eqs. (3) and (4) with respect to the normalized time $t' = tf_0$

$$h'(u, t') = f_0 \sum_{n=0}^{N-1} w_n \delta\left(t' - \frac{nu}{2}\right). \tag{6}$$

$$g'(t') = u_0 \exp\left[-\frac{\pi^2 B^2 (t'-\tau')^2}{\alpha}\right] \sin(2\pi t') \,\mathrm{step}(t'), \tag{7}$$

where $\tau' = \tau f_0 = 1.5/B$. Note that f_0 appears in Eq. (6) as an amplitude term, which does not influence the beam pattern shape.

The beam pattern of a narrow-band array varies sinusoidally with time, and it is completely represented by the angular distribution of energy. On the contrary, the far-field distribution of a wide-band array depends on t' in a more complex way. To compare it with the narrow-band case, the time dimensionality of the radiation waveform must be suppressed. We assume that the beam pattern at any observation direction u coincides with the maximum value over the time of the modulus of the acoustic pressure field [37]-[39]

$$P(u) = \max_{t'} \left| g'(t') * h'(u,t') \right|. \tag{8}$$

In the present work, we consider only symmetrical arrays, which generate symmetrical beam patterns, and, consequently, only positive observation directions ($u > 0$) are analyzed. Moreover, in the optimization process, we restrict the observation domain to $0 \leq u \leq 1$.

The accuracy and the computational efficiency of the beam pattern are important in any array synthesis method; on the other hand, a fast numerical routine is required to reduce the computational load in the iterative optimization process. We developed a very fast numerical algorithm, in which a finite set of observation directions u is considered. For each value of u, the two functions $g'(t')$ and $h'(u,t')$ are sampled with sampling rate f_s, and the convolution of Eq. (8) is evaluated. The beam pattern $P(u)$ is determined finding the maximum of the absolute value of the resulting function. In order to increase the computational speed, the sampling rate f_s is dynamically varied according to the value of u. Since the array impulse response $h'(u,t')$ is a periodic sequence with period of $u/2$, the sampling rate f_s is set as

$$f_s = m\frac{2}{u}, \tag{9}$$

where m is the lowest integer that satisfies the condition $f_s > 15$; according to this formulation, f_s is an adimensional parameter. Note that, with this choice, only the function $g(t')$ is sampled; further, the sampling rate always exceeds the value $15 f_0$, which is generally sufficient for an accurate computation of the beam pattern [40]. Finally, the numerical convolution is evaluated by a scale-shift-and-add procedure. This approach permits us to further increase the computation speed by storing all the requested samples of $g'(t')$ in the computer RAM memory, before the procedure starts. When the optimization algorithm runs, the beam pattern is calculated fetching the proper data values from the memory, avoiding to compute them at each iteration.

The proposed optimization routine considers the array elements as point sources and then does not take the element directivity function into account. After the optimization process, the pressure distribution generated by the obtained configuration is evaluated in the focal plane, taking the actual element widths into account.

III. Description of the optimization algorithm

The simulated annealing algorithm is a stochastic optimization method to solve large combinatorial problems which are similar to the thermodynamic annealing process of solids. Solutions of the optimization problem are equivalent to the states of the physical system and the cost function E is equivalent to the energy of the state. The optimization process is faced as a cooling process in which the control parameter T, which is equivalent to the temperature, is gradually lowered until a nearly perfect crystal, i.e. the global minimum, is reached. Each step of the process involves the random perturbation of the parameters and the evaluation of the energy variation. If the energy E is lowered, the perturbation is retained; if the energy is raised, the perturbation is accepted according to the Boltzman probability distribution. As the iterations go on, the temperature T is gradually lowered until the system freezes into a final state. The convergence of the simulated annealing algorithm toward a global minimum is related to the choice of the initial temperature T_{in} and to the cooling schedule. Among different implementations of the algorithm, threshold accepting has been demonstrated to be rather insensitive to the cooling schedule, yielding better results in a considerable smaller amount of computational time [28]. The major advantage of threshold accepting with respect to simulated annealing is that every new configuration, which is 'not much worse' than the old one, is accepted, without computing any probability.

We address two different problems: the optimization of the weight coefficients in a dense array and the optimization of the weight coefficients and element positions in a sparse array. In both cases, we consider the energy E as a function only of the element weights w_n; in a sparse array configuration, we assign a null weight to inactive elements. Starting from a random configuration and an initial temperature $T_{in} > 0$, we produce, at each iteration, a small perturbation of the old configuration, by varying the weight of a single element. The visiting sequence of the array elements can be random or sequential; in any case, we do not visit the same element again before all the elements have been visited once. Of course, in the sparse array optimization, both active and inactive elements ($w_n = 0$) are visited. We compute the difference ΔE between the energies corresponding to the old configuration and the new one. According to threshold accepting, we accept the new configuration if $\Delta E > -T$ (without any probability computation as in simulated annealing). The initial temperature $T_{in} > 0$ is chosen high enough to accept all the initial perturbations, even though they lead to a strong increase of the energy. As the iterations go on and when all the elements have been visited, the temperature T is gradually lowered. The threshold accepting algorithm is rather insensitive to the cooling schedule, and usually a linear scheduling is used. We set the temperature at the iteration j (which corresponds to a perturbation of the whole array) as $T(j) = 2^{-(j-1)/2}T_{in}$ so that $T(j)$ is rapidly lowered at the beginning of procedure, and it decreases more slowly at the end. We found that this cooling schedule ensures a rapid convergence toward a minimum energy state, still guaranteeing the repeatability of the achieved results.

IV. Optimization of a dense linear array

In this section, we consider an equally spaced ($\lambda/2$) wide-band array constituted by $N = 50$ elements over 25λ. The objective is to minimize the side-lobe level in the region $u_1 \leq u \leq 1$, only by varying the weight coefficients. The energy function is chosen as

$$E(w_0, w_1, \cdots w_{N-1}) = \int_{u_1}^{1} \left[\frac{P(u)}{P(0)} - \frac{P_u(u)}{P_u(0)} \right] du \tag{10}$$

where $P_u(u)$ is a reference pattern, necessary to assure the convergence of the algorithm [11], [41], which corresponds to a uniform aperture. Notice that, with this choice, the energy function can assume negative values and it reaches the minimum for an optimal configuration which has to be different from the uniform array. Even if negative energies could be meaningless in the equivalent thermodynamic annealing process, this choice lets the process freely search an array configuration with the lowest side-lobe level. Moreover, we assume that a minimum is found when the energy function E reaches a steady value which does not further vary after a certain number of iterations. Of course, this criterion does not assure the convergence toward a global minimum; anyway, we repeated the optimization experiments several times, finding nearly the same minimum energy values. In all the experiments, the initial configuration is chosen at random, and at each step only one of the weight coefficients is perturbed. In this first case, we visit all the array elements in a consecutive sequence, starting from the first and ending with the N-th. The starting point of the optimization region u_1 is a parameter of key importance, which determines the main-lobe width: lower values of u_1 yield a narrower beam width and a higher side-lobe level. Fig. 2(a) shows the optimized beam pattern of a linear array for the cases $u_1 = 0.04$ and $u_1 = 0.12$; we assume that the transmitted acoustic pulses have a $B = 70\%$ bandwidth. Fig. 2(b) shows the optimized weight coefficients as function of the element index n, which are also indicated in the first two columns of Tab. I.

Note that the optimized weight coefficients present a regular distribution, as a function of the element index n, and that their flat-top behavior indicates that no tapering is required for the elements in the central region of the array. Therefore, the optimized weight distributions resemble the Tukey window, which comprises a plateau of adjustable width [42]. The optimization process shows that the plateau width varies with u_1, i.e. varies with the starting point of the side-lobe region. The beam pattern corresponding to a Tukey window is also plotted in Fig. 2(a); comparing it with the optimized beam patterns, we can observe that the proposed approach yields better results.

We repeat the optimization procedure, reducing the variation of the weight coefficients to CTR= 5, where CTR is the Current Taper Ratio, i.e. the ratio between the maximum and the minimum value of w_n [7]. In this case, in order to enhance the stochastic-search capabilities of the algorithm, we use a random visiting sequence of the array elements. The optimized beam pattern and the corresponding weight coefficients are depicted in Fig. 3(a) and (b) respectively, for $u_1 = 0.06$ and $B = 70\%$. The third column of Tab. I reports the weight coefficients for this case. In Fig. 3(a) the beam pattern corresponding to a Gaussian tapered wide-band array is also plotted; we consider a Gaussian window function with a -14dB

truncation at the border of the aperture, which presents the same CTR. It is evident that the proposed optimization method yields a lower side-lobe level with respect to a Gaussian apodization, whereas the main-lobe width is practically the same.

We can conclude that classical window functions may result not completely satisfactory in shaping the beam pattern of wide-band arrays; on the contrary, an appropriate wide-band optimization process yields better results, since it takes into account the effect of the partial superposition of the transmitted acoustic pulses for large observation angles, where the acoustic beam spreads out in time.

V. Optimization of a sparse linear array

We start by considering an optimization problem addressed in literature under narrow-band operation [18]. Trucco et al. analyzed the problem of obtaining a beam pattern with a minimum side-lobe level by using a 'global' optimization, which also reduces the number of the active elements and the spatial aperture. Starting by an array constituted by 35 elements over 65λ, they optimized the element positions and weight coe□cients by using simulated annealing algorithm, obtaining a sparse array composed of 24 elements over 25λ. The unique constraint used was that the beam pattern must not exceed -14.4dB in the region $0.04 \leq u \leq 1$. The optimized beam pattern for narrow-band excitation is plotted in Fig. 4, and it is, as far as we know, the best result for this problem in literature. Starting from the layout optimized by Trucco et al., we compute the corresponding beam pattern for wide-band excitation, reported in the same □gure for the cases $B = 50\%$ and $B = 70\%$. It is evident that the main-lobe width remains practically unchanged under wide-band operation ($u_{-3dB} = 0.0192$), but the side-lobe level decreases (the maximum side-lobe level becomes -16.6 dB for $B = 70\%$).

In order to compare the results obtained by narrow-band and wide-band optimization, we apply the proposed wide-band optimization algorithm to the same sparse array constituted by 24 active elements distributed over a 25λ aperture ($N = 50$). The starting point of the optimization region is $u_1 = 0.04$, and we □x CTR=5.3 and $B = 70\%$. The energy function E is de□ned as

$$E(w_0, w_1, \cdots w_{N-1}) = \left\{ \int_{u \in S} \left[\frac{P(u)}{P(0)} - P_d(u) \right] du \right\}^2 \tag{11}$$

where $P_d(u) = 0.0673$ (-23.4 dB) is the desired beam pattern in the optimization region, and S is the set of the observation directions u that satisfy the relation $P(u)/P(0) > P_d(u)$. Notice that, in this case, the energy function assumes only non negative values and that the global minimum corresponds to zero. The optimized beam pattern and weight coe□cients are plotted in Fig. 5(a) and (b), respectively. The weight coe□cients are also indicated in the fourth column of Tab. I. Note that a further reduction of the side-lobe level of -6.8 dB is achieved, with respect to the narrow-band optimization, and that the main-lobe width is also slightly reduced ($u_{-3dB} = 0.0175$).

In the proposed optimization process, the beam pattern is evaluated considering the array elements as point sources. To verify the accuracy of the obtained results, we compute the acoustic pressure distribution

of a 2.5 MHz phased array with the optimized layout (positions and weights) of Fig. 5(b). The near-field pressure distribution is evaluated by using the Rayleigh-Sommerfeld diffraction formula, simplified as suggested in ref. [43]. The array consists of 24 active elements, of width $w = 0.25$ mm and height $H = 13$ mm, electronically focused at a distance $F = 80$ mm, equipped with an acoustical lens (in elevation) with the same focal length. Fig. 6 shows the transmitted pressure in the focal plane $F = 80$ mm, for $B = 70\%$, and steering angles $\theta_0 = 0°$ and $\theta_0 = 30°$. The main-lobe width is $\theta_{-3dB} = 1.33°$ without steering, and $\theta_{-3dB} = 1.61°$ for steering angle $\theta_0 = 30°$.

In order to give a further comparison with an alternative approach of wide-band sparse array design, we consider a uniformly excited sparse array constituted by 10 elements over a 18λ aperture, which has been investigated in ref. [6]. In this work, it is proved that a periodic wide-band array outperforms a random wide-band array if the array is ultrasparse, i.e. if the average interelement spacing $\bar{d}$ satisfies the condition

$$\bar{d} \geq cT. \tag{12}$$

Here T is the duration of the pulse which is approximately equal to $1/(Bf_0)$. In the previously examined arrays it is $\bar{d} = 2\lambda$, then $B = 70\%$ satisfies condition (12) and the array can be considered ultrasparse.

Fig 7 shows the beam patterns corresponding to three different configurations: dotted line refers to a randomly thinned array, dashed line to a 2λ-spaced periodic array, and the beam pattern generated by the array obtained with the proposed wide-band optimization algorithm is plotted with full line. In the latter case the positions of the active elements are $[1, 6, 9, 14, 17, 21, 24, 29, 32, 37]$, whereas, in all cases, the weights are set equal to 1. Also in this experiment, the proposed method yields better results.

Finally, we analyze the problem of a 'global' optimization under wide-band operation, which minimizes the side-lobe level and, at the same time, the number of active elements N_a. The energy function is set as

$$E(w_0, w_1, \cdots w_{N-1}) = \left\{ k_1 \int_{u \in S} \left[\frac{P(u)}{P(0)} - P_d(u) \right] du + k_2 N_a \right\}^2 \tag{13}$$

where the desired beam pattern in the region $0.04 \leq u \leq 1$ is the same as in the previous case. We fix the aperture width equal to 25λ, CTR$= 5.3$ and $B = 70\%$; moreover the optimization parameters are set as $k_1 = 100$, and $k_2 = 0.2$. Fig. 8(a) shows the beam pattern of an array constituted by only 22 active elements over 25λ. It is characterized by a main-lobe width $u_{-3dB} = 0.0179$ and a maximum side-lobe level equal to -23.3 dB. The weight coefficients corresponding to the optimized beam pattern are plotted in Fig. 8(b) and reported in the fifth column of Tab. I. Notice that all the optimized array configurations present an almost flat side-lobe distribution. Fig. 9 shows the acoustic pressure distribution of the same 2.5 MHz phased array considered above, when the weight coefficients are those of Fig. 8(b). In this case, the main-lobe width is $\theta_{-3dB} = 1.36°$ without steering and $\theta_{-3dB} = 1.68°$ for steering angle $\theta_0 = 30°$.

These results confirm the efficacy of the proposed algorithm to determine the layout of a linear array generating a desired beam pattern, under wide-band excitation.

VI. Conclusions

In the present paper, we address the problem of the optimization of linear arrays under wide-band excitation. Among the different optimization techniques, we choose the simulated annealing algorithm which has been demonstrated to be well suited for thinning arrays. In particular, we consider one of its implementations, the threshold accepting, which is rather insensitive to the cooling schedule. A simple but accurate numerical routine for the beam pattern evaluation is implemented, in order to reduce the computational load in the optimization process. For a dense linear array, we compare the proposed wide-band optimization process with two classical apodization windows, showing that the proposed approach yields a beam pattern with a lower side-lobe level, and the same main-lobe width. In the sparse arrays analysis, we compare the optimization techniques for narrow-band and wide-band arrays, showing that in the last case a further reduction of about -7dB of the maximum side-lobe level can be achieved. The proposed approach also outperforms other methods of wide-band array synthesis, as random thinning or 2λ-spaced periodic distribution. Finally, the wide-band optimization algorithm is found to be successful also for the 'global' optimization problem, where the side-lobe level and the number of elements are reduced at the same time.

In the present work, we consider wide-band ultrasonic pulses with a □xed bandwidth; in a future investigation, we intend to study the influence of the bandwidth, and of the electrical excitation waveform on the sparse array performances. Moreover, a different beam pattern formulation, such as the integrated energy over time, could lead to different results. Finally, the proposed approach can be usefully applied to reduce the number of active elements in a two-dimensional array for three-dimensional imaging, which is one of the main innovations in ultrasound imaging systems.

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Fig. 1. Linear array geometry.

Fig. 2. (a) Beam pattern of an optimized wide-band dense array constituted by 50 elements $\lambda/2$ spaced. We consider $B = 70\%$ and the both the cases $u_1 = 0.04$, and $u_1 = 0.12$. (b) Optimized weight coe□cients.

Fig. 3. (a) Beam patterns of an optimized wide-band dense array and a Gaussian windowed array. The arrays consist of 50 elements $\lambda/2$ spaced, and we consider $B = 70\%$, $u_1 = 0.06$, and CTR=5. (b) Optimized weight coe□cients and Gaussian windowed aperture, with a -14 dB truncation at the border.

Fig. 4. Beam pattern of a linear array, constituted by 24 elements over 25λ, optimized under narrow-band excitation by Trucco et al. (dotted line); beam patterns of the same array under wide-band excitation ($B = 50\%$ and $B = 70\%$) (full line).

Fig. 5. (a) Beam pattern of an optimized wide-band sparse array constituted by 24 elements over 25λ (full line); beam pattern of a uniformly excited dense array constituted by 50 elements $\lambda/2$ spaced. (b) Optimized weight coe□cients.

Fig. 6. Pressure □eld in the focal plane $Z = 80$ mm of a 2.5 MHz phased array with the optimized layout of Fig. 5(b).

Fig. 7. Beam pattern of wide-band sparse arrays. The arrays are constituted by 10 elements over 18λ aperture and the weights are equal to 1: random array (dotted line), periodic 2λ-spaced array (dashed line), and optimized array (full line).

Fig. 8. (a) Beam pattern of an optimized wide-band sparse array constituted by 22 elements over 25λ. (b) Optimized weight coe□cients.

Fig. 9. Pressure □eld in the focal plane $Z = 80$ mm