Comparison of a 3-D Richards equation-based model with the hillslope-storage Boussinesq model: A test case for nine characteristic hillslopes

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1 Introduction

A river basin is made up of interconnected hillslopes and a channel network draining these hillslopes. Both hillslopes and channels transport water to the outlet of the basin. In order to understand the hydrological processes at a catchment scale, one needs to understand the characteristic response of hillslopes and channels within the catchment. A thorough understanding of these processes is of importance for protection against natural hazards (floods, droughts), for agriculture, and for water quality management. Traditionally, hillslope response is studied using hydraulic groundwater theory. The basis of this approach (for a one-dimensional hillslope with a sloping aquifer) is described by *Boussinesq* [*1877*] and can be expressed by equation (1) presented in the next section.

No general analytical solutions for the Boussinesq equation exist. Solutions for small to intermediate slopes have been obtained through linearization [*Brutsaert* 1994; *Verhoest and Troch* 2000]. For steep slopes the kinematic wave approach becomes applicable [*Boussinesq* 1877]. Kinematic wave modeling for different slope types has been conducted by *Beven* [*1981*] and *Fan and Bras* [*1998*]. Although useful for understanding the dynamics of hillslope response, these solutions have little practical applicability since they mostly don't account for the three-dimensional soil in which these processes take place, nor for an unsaturated zone.

To describe hillslope topology, a distinction can be made between plan curvature (in the width direction of a slope) and profile curvature (in the length direction). The curvature results in three different typical shapes: concave, straight and convex. Combining three plan curvatures and three profile curvature leads to nine characteristic hillslopes: three have a convergent shape (i.e. slope width increases with distance from the bottom or outlet of the slope), three are uniform (i.e. slope width stays more or less constant) and three are divergent (i.e. slope width decreases with distance). In essence these are the hillslope shapes that one might encounter in nature.

In this study, a numerical simulation for these nine characteristic hillslopes is carried out using the three-dimensional, distributed model CATHY: a coupled model describing both subsurface flow and surface runoff. The model is capable of accurately describing flow characteristics, and thus provides us with the opportunity of studying spatially distributed phenomena like water table or storage dynamics in detail. The objective of the study is to compare the CATHY simulations with a recently developed extension to the Boussinesq model that uses the width function to account for hillslope shape [*Troch et al.* 2001 in preparation; *Paniconi et al.* 2001 in preparation]. The tasks involved in undertaking this study included:

- Generating computational grids for the nine characteristic hillslopes;
- Conducting simulations with the distributed three-dimensional CATHY model and analyzing the results.
- Comparing results from the CATHY model with those of the extended, "hillslope-storage" Boussinesq model.

The last task is especially interesting for investigating the importance of the unsaturated zone in water table dynamics and outflow characteristics, since models based on the Boussinesq equation don't account for an unsaturated zone whereas the CATHY model does.

2 Theory and Methodology

2.1 The Boussinesq equation and the hillslope-storage Boussinesq model

The Boussinesq equation

$$f\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left[kh(\cos\alpha\frac{\partial h}{\partial x} + \sin\alpha)\right] + N \tag{1}$$

is commonly used to model subsurface flow in a sloping unconfined aquifer underlain by an impermeable layer [*Childs* 1971] (Figure 1). In this equation f is the drainable porosity, $h = h(x,t)$ is the depth of the aquifer (or height of the water table) measured perpendicular to the bedrock, t is time, x is the distance along the hillslope taken parallel to the impermeable layer, k is the hydraulic conductivity, α is the slope angle, and N is an effective recharge rate or source/sink term. Equation (1) has obvious appeal because it is one-dimensional and it can be solved analytically for a wide variety of conditions [*Serrano* 1995], in particular for the drainage-only case ($N = 0$) [*Brutsaert* 1994].

There is much interest in current hydrological research to develop simple yet physically realistic models valid at the catchment scale, focusing on the subcatchment or hillslope as a fundamental unit or building block. To this end a storage-based version of the Boussinesq model has recently been proposed [*Troch et al.* 2001 in preparation], wherein, following a concept introduced by *Fan and Bras* [*1998*] and extended by *Troch et al.* [*2001 submitted*] for the kinematic wave model, the classical Boussinesq equation for idealized straight hillslopes is generalized by incorporating the width function $w(x)$ and introducing the subsurface water storage $S(x,t) = fwh$ as the dependent variable in the model. The resulting "hillslope-storage" Boussinesq model accomodates not only arbitrary plan curvature via $w(x)$, but also arbitrary profile shape by treating the width-averaged soil depth D as spatially variable in the x direction in the definition of the maximum subsurface water storage $S_c(x) = fD(x)w(x)$ (Figure 2). Thus the general features of a hillslope's plan geometry and terrain and bedrock shape, as derived for example from spatial analysis based on soil and digital elevation maps, can be accounted for. When solved numerically, spatial (and temporal) variability in recharge, boundary conditions, and conductivity are also readily handled.

The model can be used to simulate subsurface flow and storage dynamics on realistic hillslopes, and, via S_c, the surface saturation response activated by the saturation excess mechanism of runoff generation. Outflow hydrographs at the hillslope outlet or seepage face are easily partitioned between subsurface and overland flow contributions. We remark that the second mechanism for generating surface runoff — infiltration excess — can only be implicitly accounted for, in the absence of an unsaturated zone component in the Boussinesq model, by considering the recharge term N as an "effective" or actual infiltration rate and not as the potential rate represented by the rainfall amount. Addition of the source/sink term N to equation (1) extends the range of applicability of the Boussinesq model from drainage studies to storm-interstorm simulations [*Verhoest and Troch* 2000].

Combining the storage-based continuity equation

$$\frac{\partial S}{\partial t} = -\frac{\partial Q}{\partial x} + Nw \tag{2}$$

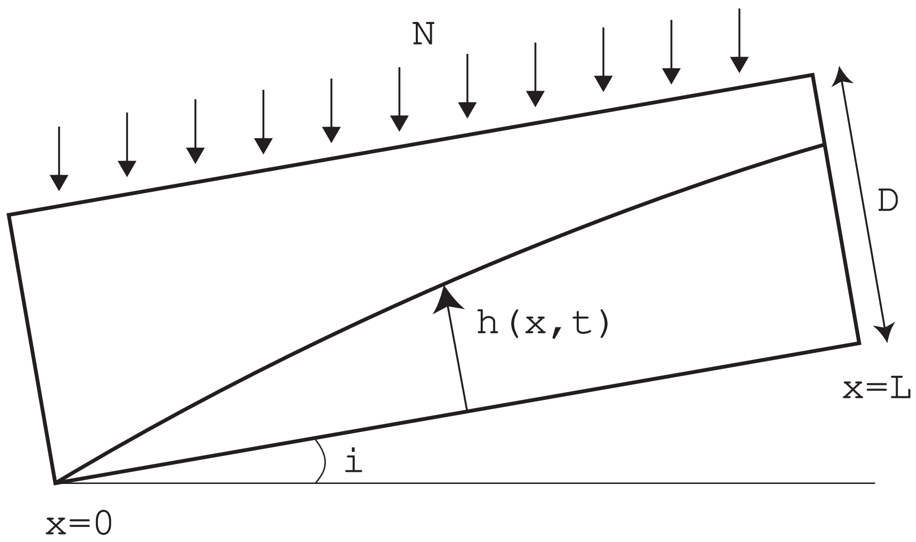

Figure 1: Schematic of a simplified straight hillslope representing a sloping unconfined aquifer underlain by an impermeable layer.

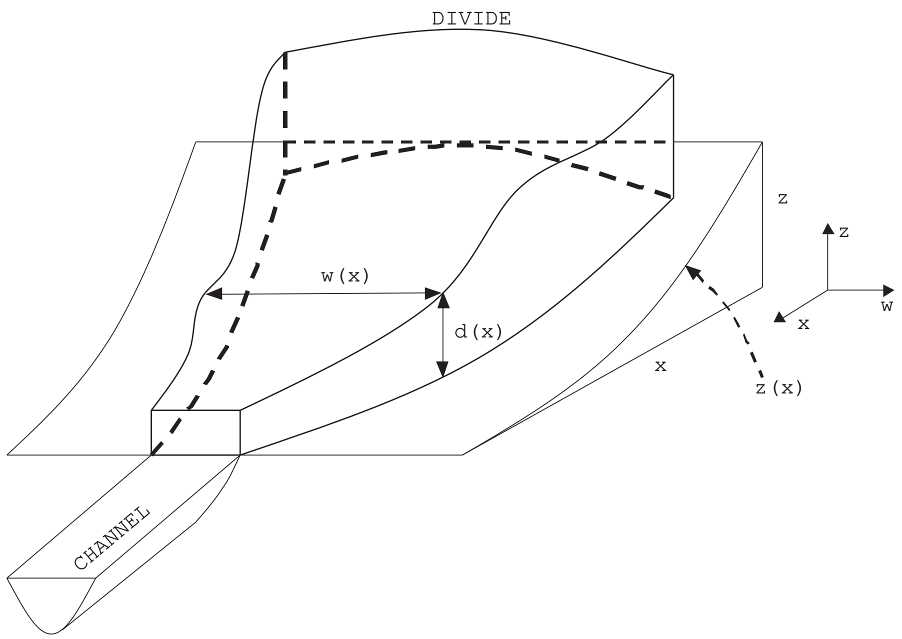

Figure 2: Schematic of a more general three-dimensional hillslope.

with Darcy's law for a hillslope with width function $w(x)$

$$Q = qw = -kh(\cos\alpha \frac{\partial h}{\partial x} + \sin\alpha)w \quad (3)$$

and substituting S/fw for h in (3) we obtain the hillslope-storage Boussinesq equation [*Paniconi et al.* 2001]

$$f\frac{\partial S}{\partial t} = \frac{k\cos\alpha}{f}\frac{\partial}{\partial x}\left[\frac{S}{w}(\frac{\partial S}{\partial x} - \frac{S}{w}\frac{\partial w}{\partial x})\right] + k\sin\alpha\frac{\partial S}{\partial x} + fNw \quad (4)$$

where Q is a volumetric discharge flux and q is the Darcy flux for a sloping unconfined aquifer of unit width. Expanding the second order derivative term in (4) gives

$$f\frac{\partial S}{\partial t} = \frac{k\cos\alpha}{fw}\left[(\frac{\partial S}{\partial x})^2 + S\frac{\partial^2 S}{\partial x^2} - \frac{3S}{w}\frac{\partial S}{\partial x}\frac{\partial w}{\partial x} + \frac{2S^2}{w^2}(\frac{\partial w}{\partial x})^2 - \frac{S^2}{w}\frac{\partial^2 w}{\partial x^2}\right] + k\sin\alpha\frac{\partial S}{\partial x} + fNw \quad (5)$$

Dropping the three terms containing $\partial w/\partial x$ yields the simplified form of the hillslope-storage Boussinesq model

$$f\frac{\partial S}{\partial t} = \frac{k\cos\alpha}{fw}\left[(\frac{\partial S}{\partial x})^2 + S\frac{\partial^2 S}{\partial x^2}\right] + k\sin\alpha\frac{\partial S}{\partial x} + fNw \quad (6)$$

2.2 Characteristic hillslopes

The nine characteristic hillslopes used in this hydrological study are particularly suited for model intercomparison and testing of numerical properties. The three-dimensional shape of the hillslopes is described analytically. Analytical descriptions have several advantages over gridded approximations of surface shape when dealing with model intercomparison. In the first place, an analytical description enables a consistent translation between quasi-3D hillslope representation (using a function describing hillslope width for example) and a fully-3D representation. Secondly, the analytical description allow us to easily generate gridded as well as triangulated surface meshes at any regular or irregular resolution. This latter property is convenient for experimentation with grid size and intercomparison of finite element and finite difference models, for example.

The hillslopes are characterised by the combined curvature in the length direction (profile curvature) and the curvature in the width direction (plan curvature). This description gives us three possible shapes for profile as well as plan: concave, straight, and convex. Combining plan and profile curvature leads to nine characteristic hillslopes. The equation describing the hillslope surface shape reads:

$$z(x,y) = E + H(1 - \frac{x}{L})^n + aw^2 \quad (7)$$

where

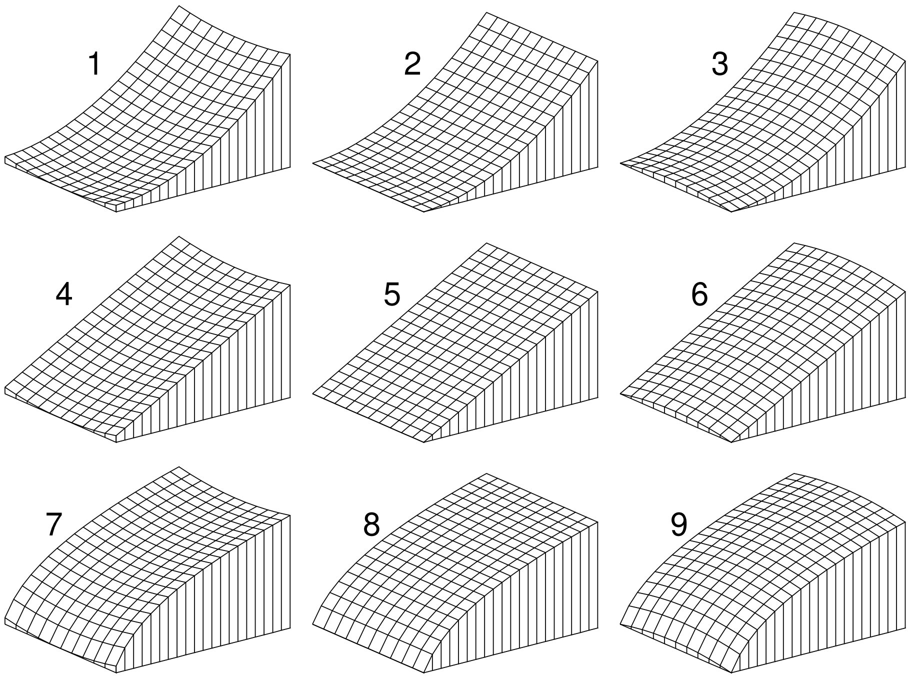

Figure 3: 3-D view of the nine characteristic hillslopes.

z	[L]	= the elevation above a reference point
x	[L]	= slope length
w	[L]	= slope width
L	[L]	= slope length parameter
E	[L]	= the elevation at point x = L
H	[L]	= height difference between x = 0 and x = L
n	[-]	= profile curvature parameter (n > 0)
a	[1/L]	= plan curvature parameter
A	[L^2]	= hillslope surface area

Note that for n<1 the profile curvature curvature is convex, for n=1 it is straight, and for n>1 it is concave. For a<0, the plan curvature is convex, for a=0 it is straight, and for a>0 it is concave.

The parameters used to generate the nine characteristic hillslopes are listed in Table 1. The hillslopes are depicted in Figure 3. The numbers in the figure refer to Table 1.

The profile curvature is important because it reflects the change in slope angle and thus controls change of velocity of mass flowing down along the slope curve. The plan curvature reflects the change in aspect angle and determines the divergence or convergence of water flow. Thus, both plan and profile determine the location of the slope divides and consequently the slope width.

Now a distinction can be made between three different hillslope shapes: convergent, uniform, and divergent. For convergent hillslopes (concave plan shape) the slope width increases from $x = 0$ to $x = L$, for uniform hillslopes (straight plan shape) it is constant and for divergent

Table 1: Parameter values for the nine characteristic hillslopes.

Hillslope #	profile	plan	H	n	$a(\cdot 10^{-4})$	L	A
1	concave	concave	5.01	2.00	5	105	2496
2	concave	straight	5.01	2.00	0	105	5000
3	concave	convex	5.01	2.00	-5	105	646
4	straight	concave	5.25	1.00	5	105	2160
5	straight	straight	5.25	1.00	0	105	5000
6	straight	convex	5.25	1.00	-5	105	2161
7	convex	concave	8.16	0.31	5	105	1410
8	convex	straight	8.16	0.31	0	105	5000
9	convex	convex	8.16	0.31	-5	105	2386

hillslopes (convex plan shape) it decreases as one moves up the slope profile. In order to find the precise location of the slope divides, a gradient descent (for convergent and uniform hillslopes) or gradient ascent (for divergent hillslopes) of the hillslope is performed starting at the sides of the hillslope. The hillslopes are constructed in such a way that divergent hillslopes are widest at the outlet, convergent hillslopes are widest at the crest and uniform hillslopes have a constant slope width. This is illustrated in Figure 4, which depicts a top view (view of the xw-plane) with contour lines and slope divides.

Since the hillslopes are defined to that part of the slope which lies inside the divides, the surface area is no longer 5000 m^2, but varies from one hillslopes to another. The values of surface area are listed in Table 1.

At this moment the hillslope-storage Boussinesq model cannot account for surface or bedrock curvature. For the model intercomparison in this study, therefore the bedrock and surface shape are straightend (i.e. curvature is set to 0). The divides of the slopes, based on profile and plan curvature, remain the same however. A triangular network is created based on these slope shapes and slope divides. A three-dimensional figure of these triangular networks is shown in Figure 5. By straightening the nine hillslopes, all initial slopes with a straight plan get the same three-dimensional shape. This reduces the number of different hillslopes to 7.

The numerical grid consists of 1407 surface nodes: 201 nodes in the length direction (x-direction) and 7 nodes in the width direction (w-direction). This results in 2400 surface triangles. The CATHY model, to be described in detail in the next section, generates a three-dimensional tetrahedral network on the basis of this triangular network. For a 20-layer vertical dicretisation this results in a total of 29547 nodes in the three-dimensional network. Because of the complex non-linear relations between soil moisture content on the one side and conductivity and pressure head one the other side, the vertical dicretization is finer near the surface and coarse towards the bottom layer. The 'ZRATIO' ,describing the relative contribution of a soil layer to the total soil depth, has a value of 0.03 for the 10 top layers and 0.07 for the 10 bottom layers.

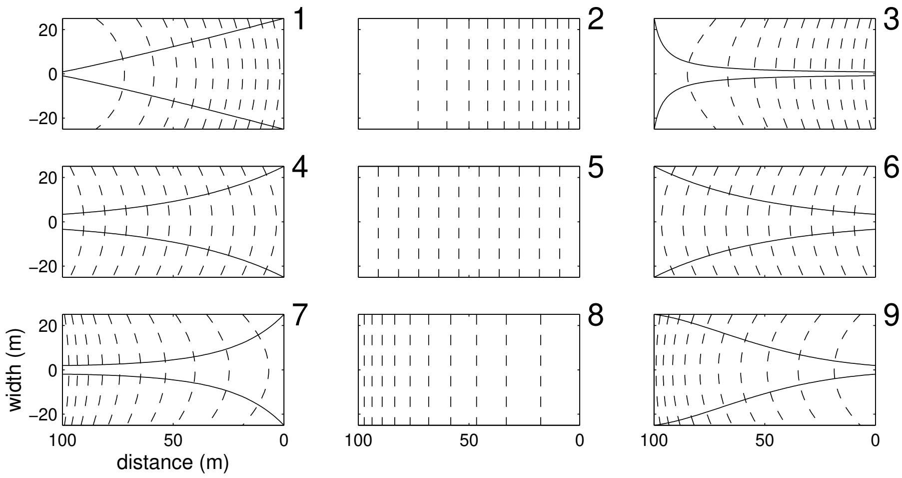

Figure 4: Contour lines and slope divides for the nine characteristic hillslopes.

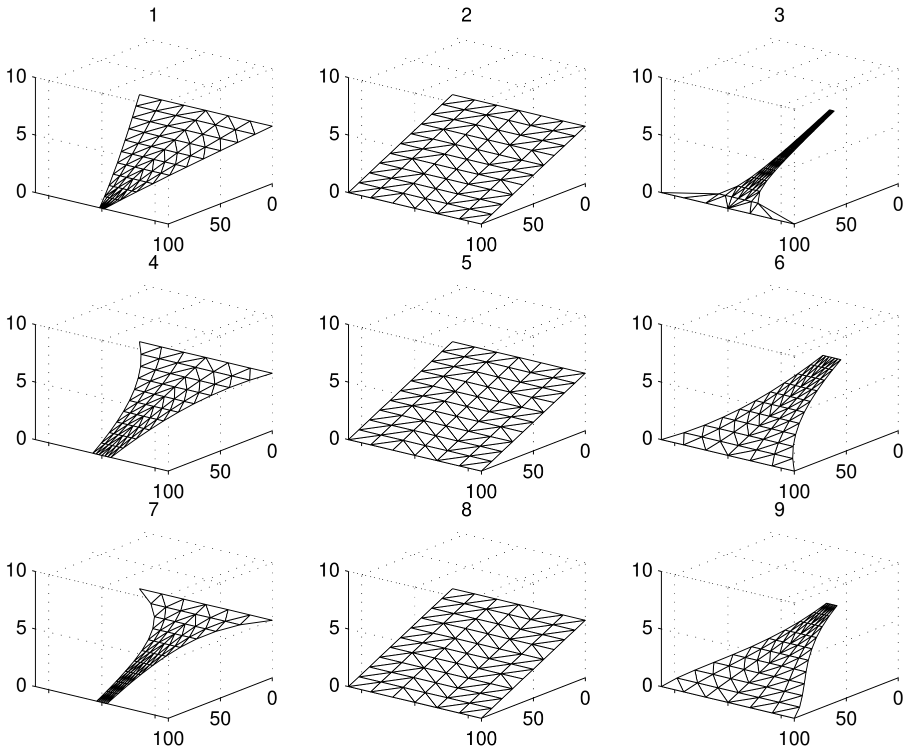

Figure 5: 3-D view of the nine characteristic hillslopes and their slope divides (for visual clarity a coarser mesh is shown than the one used in the simulations).

2.3 Hillslope settings for the test case simulations

To investigate the flow processes along hillslopes, simulations are carried out for the nine characteristic hillslopes just described. The general setup of these hillslopes for the test case simulations is:

- The hillslopes are 100 meters in length and have a slope width depending on the slope shape (maximum width value is 50 meters).
- The height difference between hillslope crest and outlet is 5 meters and soil depth is set to 2 meters for all slopes.
- During the simulation there is only drainage with no atmospheric input (i.e. no precipitation or evaporation). Simulations with rainfall recharge will be performed in a follow-up study.
- The initial condition for the Richards equation model is a situation of vertical hydrostatic partial saturation, with the water table positioned at 0.40 m above the bedrock.
- Simulations are conducted for a "sandy" soil type.

Boundary conditions and a more detailed description of the grids is given later.

2.4 The 3-D distributed model CATHY

In this study the three-dimensional distributed finite element model 'CATHY' (CATchment HYdrology) is applied. CATHY is a physically based three-dimensional distributed model built up out of a subsurface flow module (FLOW3D) based on Richards' equation [*Paniconi and Wood* 1993; *Paniconi and Putti* 1994] coupled to a surface routing module (SURF_ROUTE) based on a diffusion wave approximation [*Orlandini and Rosso* 1996]. The numerical scheme uses a finite element Richards' equation solver and a surface DEM-based finite difference module. The model can be described by a system of two partial differential equations [*Bixio et al.* 2000]:

$$\sigma(S_w)\frac{\partial \psi}{\partial t} = \nabla \cdot [K_s K_r(S_w)\ (\nabla \psi + \eta_z)] + q_s(h) \quad (8)$$

$$\frac{\partial Q}{\partial t} + c_k \frac{\partial Q}{\partial s} = D_h \frac{\partial^2 Q}{\partial s^2} + c_k q_L(h, \psi) \quad (9)$$

where

$$\sigma(S_w) = S_w S_s + \phi \frac{\partial S_w}{\partial \psi} \quad (10)$$

$$S_w(\psi) = \frac{\theta}{\phi} \quad (11)$$

and θ is volumetric water content, ϕ is porosity, $S_w(\psi)$ is water saturation, S_s is the aquifer specific storage coefficient, ψ is pressure head, t is time, ∇ is the gradient operator, K_s is the saturated hydraulic conductivity tensor, $K_r(S_w)$ is the relative hydraulic conductivity function, $\eta_z = (0,0,1)^T$, z is the vertical coordinate directed upward, and q_s represents distributed source (positive) or sink (negative) terms (volumetric flow rate per unit volume).

The surface water is routed using (9) along each single hillslope or channel link using a one-dimensional coordinate system s defined on the drainage network. In this equation, Q is the discharge along the channel link, c_k is the kinematic wave celerity, D_h is the hydraulic diffusivity, and q_L is the inflow (positive) or outflow (negative) rate from the subsurface into the cell, i.e., the overland flow rate. We note that q_s [L^3/L^3T] and q_L [L^3/LT] are both functions of the ponding head h, and that h can be easily derived from the discharge Q via mass balance calculations.

This system of equations must be solved simultaneously for the unknown vector (Q, ψ) or (h, ψ). Nonlinearities arise in the $S_w(\psi)$ and $K_r(S_w)$ characteristic curves in the Richards' equation, in the nonlinear dependence of q_s on the ponding head, and in the nonlinear dependence of q_L on ψ.

In the subsurface module, the relative hydraulic conductivity function can be calculated using the expressions descibed by *van Genuchten and Nielsen* [*1985*] ,*Brooks and Corey* [*1964*], or *Huyakorn et al.* [*1984*]. The nonlinearities in (8) are linearized using either Picard or Newton type iteration [*Paniconi and Putti* 1994]. The three-dimensional spatial discretisation is done using tetrahedral elements and linear basis functions. For the discretisation in time a finite difference scheme is used.

In the surface routing module a one-dimensional representation of surface runoff is given in (9). The assumpion is made that surface flow will concentrate in rills or rivulets that are formed by local irregularities or erosion processes. A routing scheme based on the Muskingum-Cunge method, with different distribution of Gauchler-Strickler coefficients, is used to describe these channel flows.

The code handles temporally and spatially variable boundary conditions, including seepage faces, atmospheric inputs and heterogeneous material properties (such as hydraulic conductivity for example).

In this study, the CATHY model was run in subsurface mode only in order to make the results compatible with those of similar simulations ran using the hillslope-storage Boussinesq model. As soon as exfiltration occurs (in these simulations only due to saturation excess) the excess water is removed and not routed. Thus reinfiltration does not occur.

2.5 Soil properties

The soil type used in the simulations can be classified as "sandy", and the Brooks-Corey [*Brooks and Corey* 1964] relationship was used to describe the nonlinear $\theta(\psi)$ and $K_r(\psi)$ relationships.

$$\theta(\psi) = \theta_r + (\theta_s - \theta_r)\left(\frac{\psi_c}{\psi}\right)^{\beta} \qquad \psi \le \psi_c \tag{12}$$

$$\theta(\psi) = \theta_s \qquad \psi > \psi_c \tag{13}$$

$$K_r(\psi) = \left(\frac{\psi_c}{\psi}\right)^{2+3\beta} \qquad \psi \le \psi_c \tag{14}$$

$$K_r(\psi) = 1 \qquad \psi > \psi_c \tag{15}$$

where θ is volumetric water content, β is a pore size distribution index, θ_s is the saturated moisture content (and is equal to porosity for a sandy soil), θ_r is the residual moisture

Table 2: Soil parameters for the test simulations.

S_s	aquifer specific storage	0.01
ϕ	porosity	0.30
K_s	saturated hydraulic conductivity	1.0 m/hr (2.78×10^{-4} m/s
β	Brooks-Corey parameter	3.3
θ_r	residual moisture content	0.0
ψ_c	air entry pressure head (capillary fringe)	-0.25 m and -0.12 m

capacity, and ψ_c is the air entry pressure head value or capillary fringe value. The values for these parameters are given in Table 2.

2.6 Boundary and initial conditions

The slope divides (slope sides as well as the slope crest) of the nine hillslopes are considered to be zero-flux boundaries (i.e., Neumann-type condition), except for the nodes in the bottom layer at the outlet; here pressure head is set to 0 m (i.e., Dirichlet-type condition).

The simulations are started using hydrostatic partial saturation in the vertical direction. A water table height of 0.4 m above the bedrock is chosen. This results in a pressure head of -1.6 m at the surface (since soil depth is set to 2 m). For the nodes at the bedrock, initial pressure head values are 0.4 m. These initial conditions were used instead of full saturation to avoid occurrence of surface runoff, given that the study is focused on comparing the subsurface response of the two models.

2.7 Boussinesq model simulations

The simulations with the hillslope-storage Boussinesq model are carried out for the same hillslopes as decribed in section 2.2. As initial condition a water table height of 0.4 m above the bedrock was used. The difference with the initial condition for CATHY is that for the Boussinesq case, with no representation of the unsaturated zone, there is no storage of water above 0.4 m. Boundary conditions as well as saturated conductivity are exactly the same as for the CATHY model.

The drainable porosity parameter f of equation (1) is normally calculated as:

$$f = \frac{V_c}{V_i} \tag{16}$$

where V_c is the total (steady state) cumulative flow volume drained during the CATHY simulations $[m^3]$ and V_i is total initial soil volume $[m^3]$ in a fully saturated hillslope. However the initial condition in our runs is not full saturation, but a water table height of 0.40 m above the bedrock. Since soil depth is over 0.40 m, for the CATHY model there is additional water in the unsaturated zone, most notabley from the capillary fringe, which must be taken into account as well. In this study the drainable porosity is therefore calculated as:

$$f = \phi\frac{V_c}{V_i} \tag{17}$$

Table 3: Porosity values for hillslope-storage Boussinesq simulations

Hillslope #	V_i	f	V_i	f
1	559.5	0.297	415.3	0.294
2	1121.0	0.293	832.0	0.296
3	114.9	0.270	107.5	0.286
4	484.3	0.297	359.5	0.298
5	1121.0	0.293	832.0	0.296
6	484.5	0.287	359.6	0.299
7	316.0	0.297	234.6	0.294
8	1121.0	0.293	832.0	0.296
9	534.9	0.289	397.0	0.300

where ϕ is porosity and V_i is total initial water volume for CATHY [m^3] instead of total soil volume.

Since the pressure head distribution at $t = 0$ is known, the initial water volume can be calculated using equations (13) and (14) for any given x, w-coordinate. Since the distribution is uniform over the hillslope, one value can be calculated and then be multiplied by the surface area of the hillslope (see Table 1). The values of initial water volume and f are listed in Table 3. Two sets of V_i and f values are listed: the first are values using $\psi_c = -0.25$ m, the second using $\psi_c = -0.12$ m.

3 Results

3.1 CATHY water tables

Because simulations have to be carried out until steady state is reached (for calculating drainable porosity), the time of simulation is set to 10^{10} seconds (approximately 115740 days or 320 years). The computational time needed for these simulations is not excessive, because as the system gets closer to steady state, timesteps become very large. Outputs are determined at (approximately) $0, 1, 2, 5, 10, 20, 50, 100$ and 115740 days. Relative water table height or relative storage (water table height/soil depth) of the nine different slopes are depicted in Figure 6. The water table at $t = 0$ is at 0.40 m above bedrock and soil depth is 2 m for all slopes. For the first of the nine hillslopes the time of output of the water table is indicated, to clearify the chronology.

We can see that for none of the hillslopes surface runoff occurs. As could be expected there is quite a difference in response speed between the slopes. For example slope number 1 still has a saturated part after 50 days, whereas slope number 3 is completely unsaturated after 10 days. This difference is caused by slope shape: because of the convergent shape of slope numbers $1, 4$ and 7, soil moisture in the upslope parts of these slopes moves downslope more easily than soil moisture downslope. This causes water to accumulate near the outlet and makes water tables rise in these places. This phenomenon is strongest for slope number 1, the most convergent slope. For all three convergent slopes, we can see a rising of water table above its initial value of 0.2.

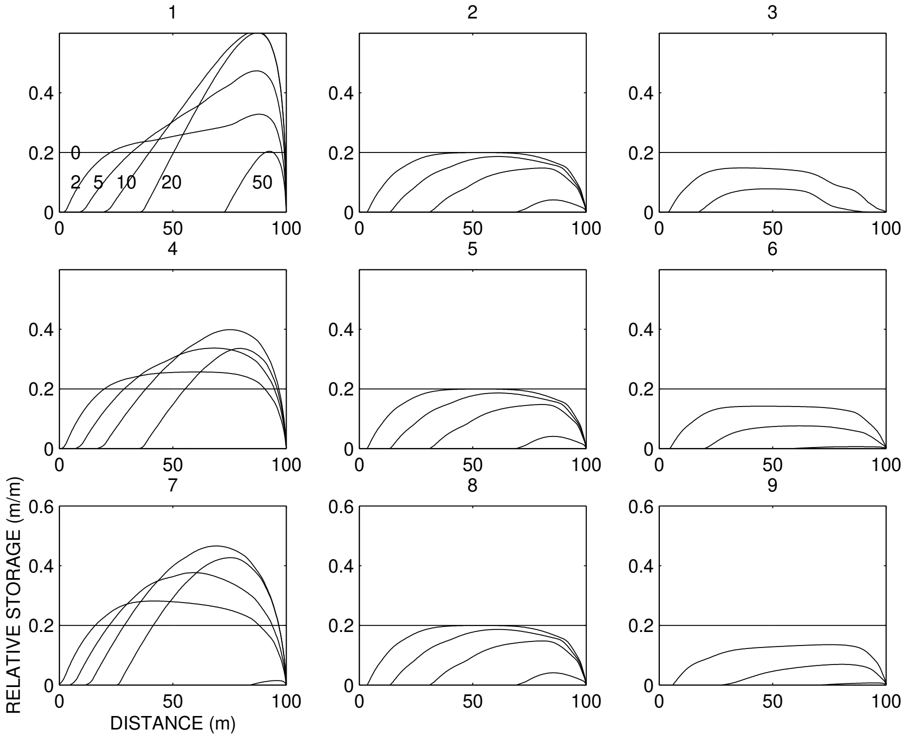

Figure 6: Relative water table height at $0, 2, 5, 10, 20$ and 50 days for the CATHY simulations of the nine characteristic hillslopes.

This is not the case for slope numbers $2, 5$ and 8: these slopes have no 'bottleneck' downslope and thus the water table and the "saturation wave" gradually recede in time. Because the slopes have the same shape (due to straightening of the slopes), they exhibit exactly the same response. The divergent slopes $3, 6$ and 9 show a rapidly dropping water table in time. Because of the divergent slope shape the soil moisture doesn't accumulate but disperses. This makes the shape of the water table look 'stretched'. This effect is strongest for slope number 3, which has a relatively large outlet surface in relation to total slope surface, giving it the ability to drain very rapidly. This is the only slope that is completely unsaturated after 10 days.

3.2 CATHY hydrographs and cumulative flow

In Figure 7 the hydrographs [mm/day] for the nine different slopes are depicted. The outflow rate [m^3/day] is divided by hillslope surface area [m^2] to avoid the scale differences caused by differences in surface area. Figure 7 is strongly related to Figure 6 in the sense that the times for which the water table is highest in Figure 6 coincide with peak flows in the hydrographs of Figure 7. Furthermore hillslopes with a rapidly dropping water table in time (i.e., divergent slopes) show a fast drop in subsurface outflow as might be expected. The opposite holds for the convergent slopes: first a rising of the water table above the inital value can be seen (see Figure 6), and later it drops. The same pattern can be seen in the hydrographs of Figure 7 . For the straight hillslopes $2, 5$ and 8, a more gradual decrease in outflow rate is shown, just as the water table drops gradually.

In Figure 8 the cumulative flow [mm] for the nine different slopes is depicted. Also here

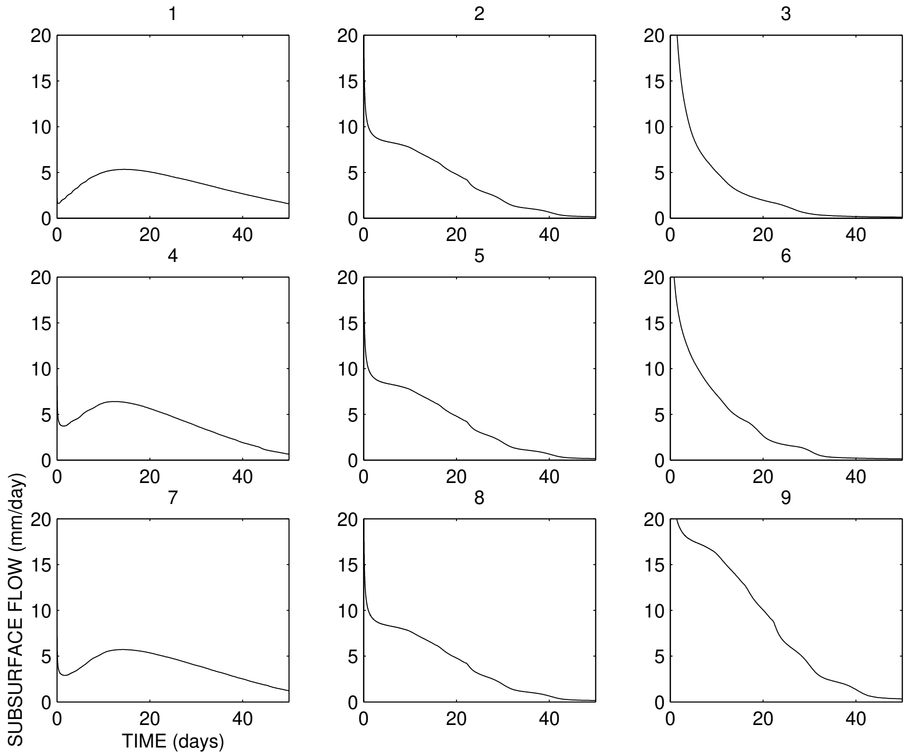

Figure 7: Outflow hydrographs for the CATHY simulations of the nine characteristic hillslopes.

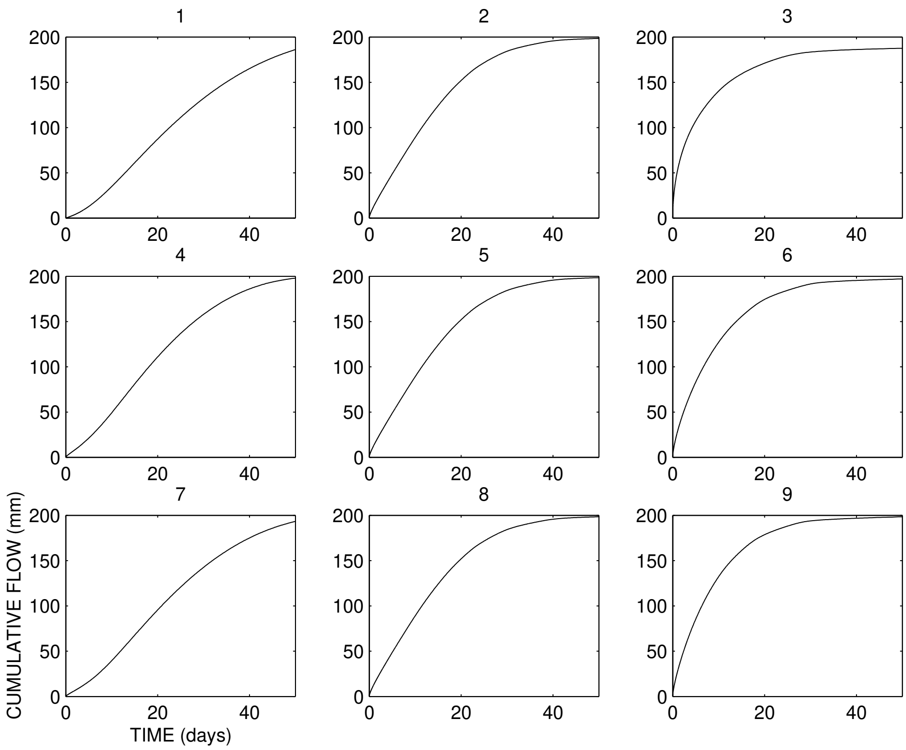

Figure 8: Cumulative flow hydrographs for the CATHY simulations of the nine characteristic hillslopes.

Table 4: CATHY simulation statistics.

Hillslope #	MBE $[m^3]$	RMBE [%]	BSTEP	TSTEP	SIMTIME $[s]$
1	7.5	-1.4	89	1784	4988
2	12.4	-1.1	111	1768	3525
3	1.7	-1.3	115	1804	3339
4	5.3	-1.1	86	1665	3877
5	12.4	-1.1	111	1768	3525
6	3.8	-0.8	102	1678	2903
7	4.3	-1.4	93	1813	4864
8	12.4	-1.1	111	1768	3525
9	4.7	-0.9	85	1519	2697

the cumulative flowvolume [m^3] is divided by hillslope surface area [m^2] to make the plots intercomparable. As might be expected based on Figures 6 and 7, the convergent hillslopes show a slow drainage, the straight hillslopes show a gradual drainage and the divergent hillslopes show a fast drainage, due to hillslope shape. This can be seen in the cumulative flow plots depicted in Figure 8. Furthermore we can see in Figure 8 that all hillslopes tend to drain the same cumulative amount of water, normalized with respect to surface area. This is to be expected given that the initial water table position is the same for all nine hillslopes. The small differences in cumulative drainage are due to the varying effects of the unsaturated zone for the differently-shaped hillslopes. By the end of the simulation all hillslopes drained between 90% and 99% of their initial water volume.

3.3 CATHY simulation statistics

A brief overview of the simulation statistics (simulation time, mass balance error and backstepping) for each hillslope is given. The results are depicted in Table 4. In Table 4 MBE stands for "mass balance error", RMBE is "relative mass balance error", BSTEP is "number of backstepping occurrences", TSTEP is "number of timesteps" and SIMTIME is "total CPU time used for the simulation".

From the statistics one can see that the simulation results are acceptably accurate (small mass balance errors) and calculation times are not excessive (the simulations were run on an 800 MHz Linux PC).

3.4 Comparison with the hillslope-storage Boussinesq model

The results of the CATHY simulations will now be compared with those of the hillslope-storage Boussinesq model. As stated before, the Boussinesq model describes saturated flow parallel to the bedrock, without describing processes that take place in the unsaturated zone. In Figure 9 hydrographs of both the CATHY and Boussinesq model are depicted.

One can notice that the hydrographs are most alike in the case of the convergent slope shape (hillslope numbers 1, 4, and 7). The general shape of the hydrographs is almost exactly the same for both models. The CATHY output however has a higher value. This is caused by the

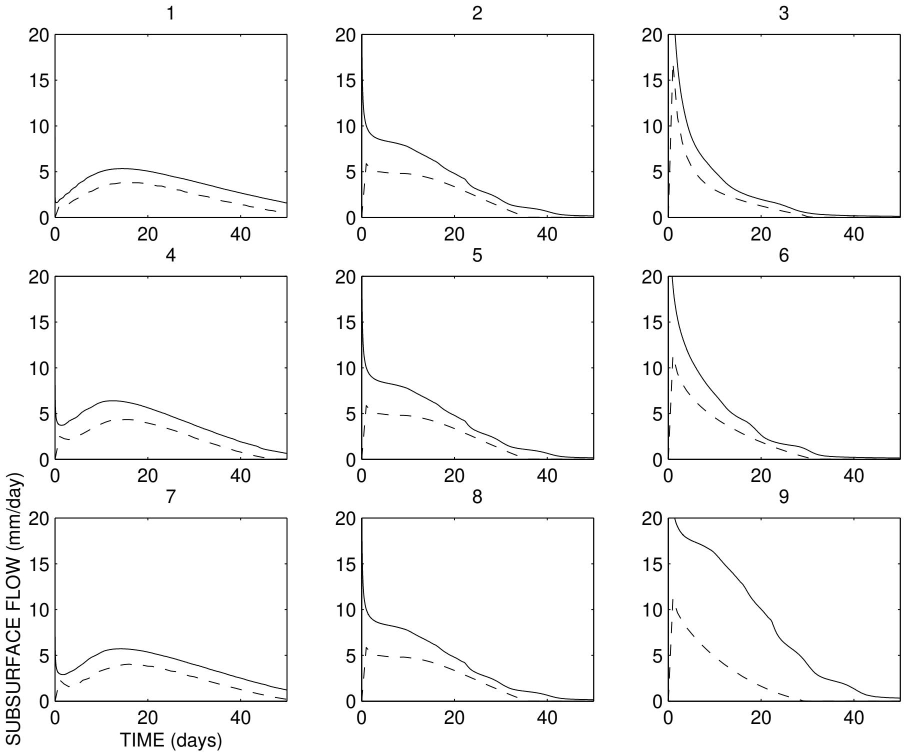

Figure 9: Outflow hydrographs of the CATHY and hillslope-storage Boussinesq models (solid = CATHY, dashed = Boussinesq) with $\psi_c = -0.25$ m for the CATHY model.

fact that the initial state for the CATHY simulations is not exactly the same as the initial state in the Boussinesq case, due to the presence of a capillary fringe and an unsaturated zone in the CATHY model. This means that for CATHY simulations there is more water initially present, causing extra outflow.

Given that a capillary fringe value of 0.25 m is quite high relative to a water table height of 0.4 m, and thus a relatively large amount of water is stored in the soil just above the saturated zone, this probably has a larger effect than the unsaturated zone conditions.

For the uniform slopes $(2, 5$ and $8)$ the hydrographs are further apart than for the convergent slopes. With increasing time however we can see that the the two hydrographs converge towards eachother. For the divergent slopes 3 and 6 we can see that both models show the same type of drainage behavior, but the outflow is higher for the CATHY model. For (divergent) hillslope number 9 the two hydrographs are very different however, for reasons that are not yet clear. [It was subsequently discovered that this anomalous difference was caused by a small typo in one of the MATLAB scripts used to plot the results].

The good match between the Boussinesq and CATHY outflow hydrographs for convergent slopes may be favored in part by the fact that these slopes drain more slowly than the straight and divergent ones, thus remaining relatively more saturated, thereby reducing the impact of the unsaturated zone.

In Figure 10 the water table values for the CATHY model and the hillslope-storage Boussinesq model are displayed. In this figure we can see that water tables for both models also show a lot of similarity in the case of the convergent hillslopes, where the match for straight and

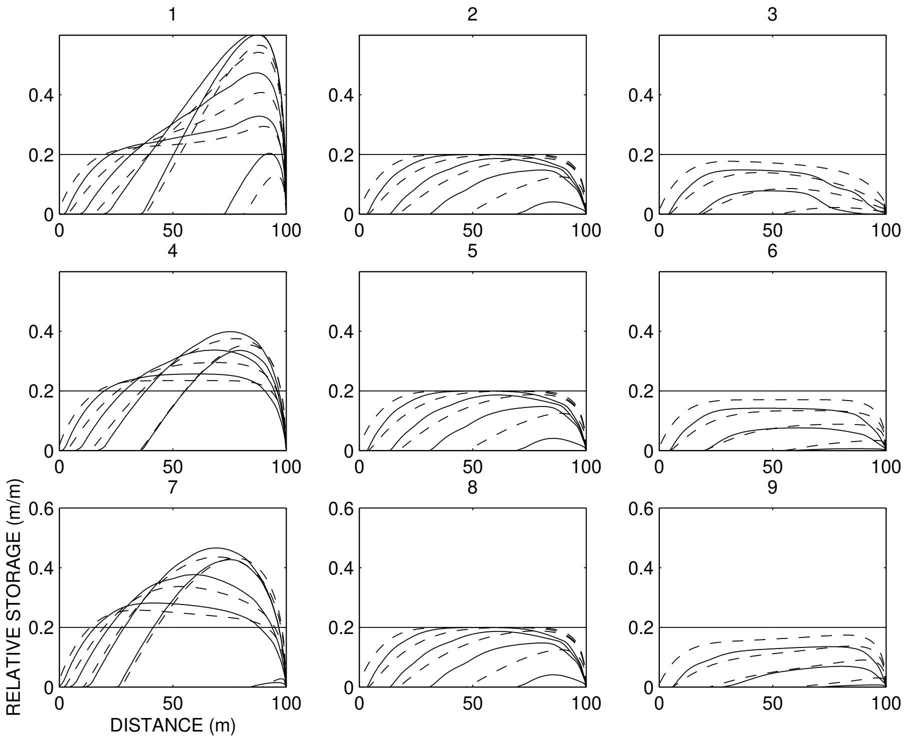

Figure 10: Relative water table height at 0,2,5,10,20 and 50 days for the CATHY and hillslope-storage Boussinesq models (solid = CATHY, dashed = Boussinesq) with $\psi_c = -0.25$ m for the CATHY model.

divergent slopes is again poorer. For the faster draining straight and divergent hillslopes for which the unsaturated zone plays a relatively more important role as described above, the impact of the unsaturated zone storage is to allow for faster water table drops than for the Boussinesq model, for which the same amount must drain from a smaller (saturated) soil depth.

Since the capillary fringe appears to be of significant importance, additional simulations will now be conducted with a lower ψ_c value.

3.5 Comparison with the hillslope-storage Boussinesq model using a reduced ψ_c value for CATHY

The hydrographs of the simulations using $\psi_c = -0.12$ m in the CATHY model are displayed in Figure 11, where the results are compared to the same Boussinesq runs of Figures 9 and 10.

In Figure 11 it can be seen that the outflow values for the CATHY model have decreased as expected and now match much more closely those of the Boussinesq model. Hillslope number 9 is still the only slope for which the results of both models differ substantially [but see previous note], although even for this slope the hydrographs of both models are now closer. It should be expected that for a further decrease of ψ_c the results for all hillslopes will get even closer, but conducting these simulations was not possible as will be discussed in the next section.

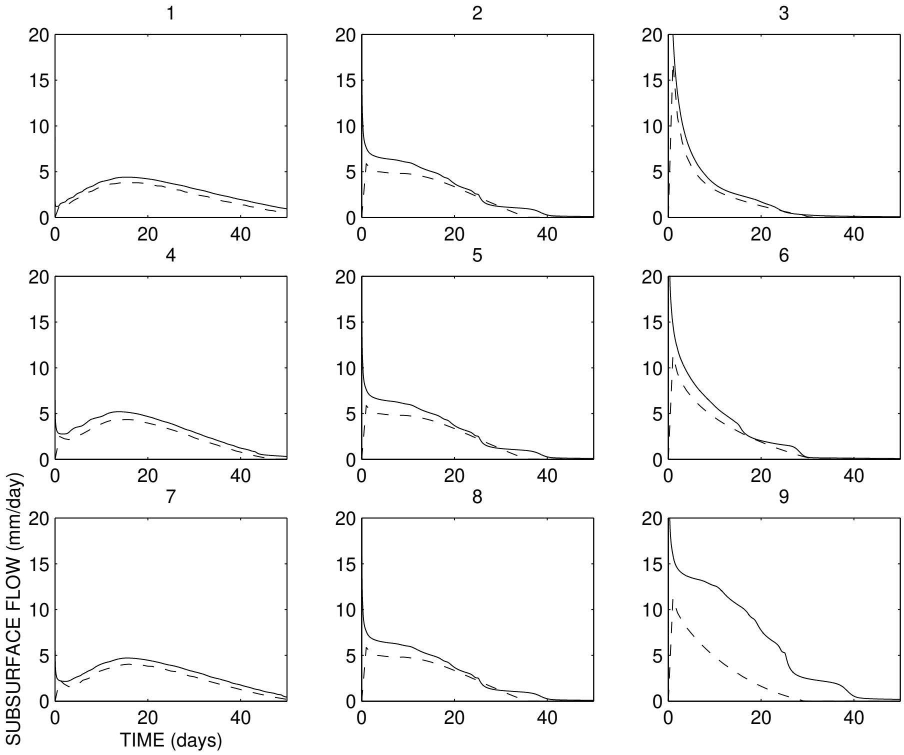

Figure 11: Outflow hydrographs of the CATHY and hillslope-storage Boussinesq models (solid = CATHY, dashed = Boussinesq) with $\psi_c = -0.12$ m for the CATHY model.

In Figure 12 the relative water table height over the slope profile is plotted at different times. This figure can be compared to the (similar) plot depicted in Figure 10. The water tables now show very good agreement between the two models, especially for the convergent slopes.

3.6 CATHY simulation statistics using a reduced ψ_c value

As in Table 4, we give in Table 5 an overview of the numerical performance of the CATHY model, this time for the runs using a reduced capillary fringe value. Although the global mass balance errors are still acceptabley small, we see that compared to the runs with $\psi_c = -0.25$ m the CATHY model had more numerical problems for all the hillslopes, with all summary statistics higher (worse) than in Table 4. The reason for this can be seen from Figures 13 and 14. As ψ_c is reduced with all other Brooks-Corey parameters kept fixed, the relationships given by equations (13) to (15) become decidedly steeper and yield much drier soils over the same range of pressure heads. Such sharp-gradient dry-soil conditions make the problem extremely nonlinear and quite difficult to solve numerically. That this is so can also be seen from the "less smooth" outflow and water table curves of Figures 11 and 12 compared to Figures 9 and 10, and also from the fact that, for all four of these figures, the curves become "less smooth" for those hillslopes with a relatively more important unsaturated zone (smoothest for convergent hillslopes and least so for divergent ones).

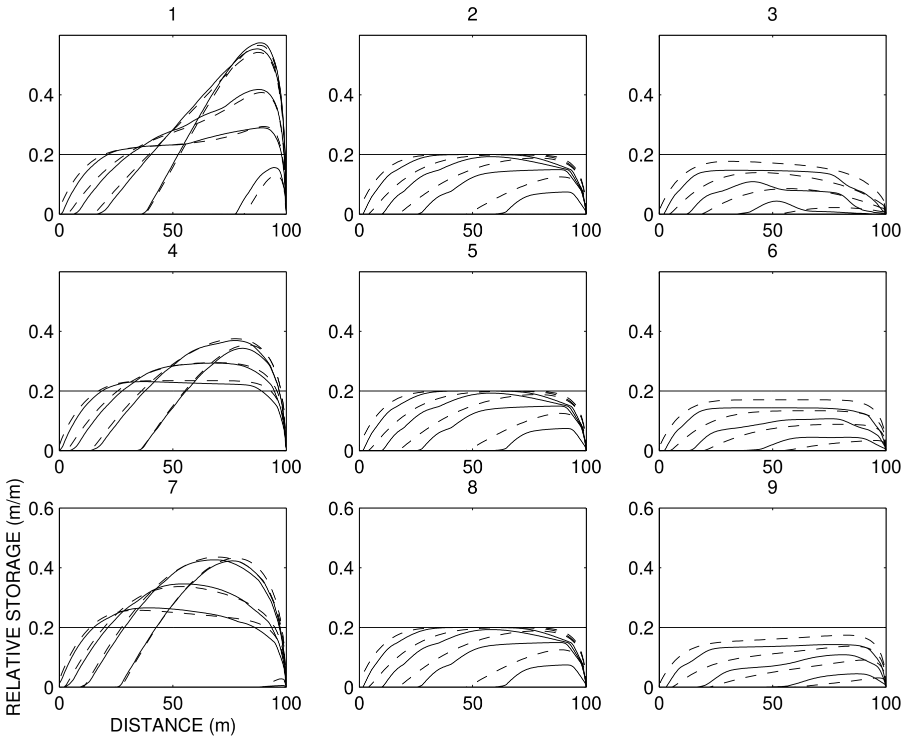

Figure 12: Relative water table height at 0,2,5,10,20 and 50 days for the CATHY and hillslope-storage Boussinesq models (solid = CATHY, dashed = Boussinesq) with $\psi_c = -0.12$ m for the CATHY model.

Table 5: CATHY simulation statistics using a reduced capillary fringe value.

Hillslope #	MBE $[m^3]$	RMBE [%]	BSTEP	TSTEP	SIMTIME $[s]$
1	12.3	-3.0	271	4574	11735
2	14.3	-1.7	321	3926	7337
3	2.4	-2.4	293	3536	5842
4	6.6	-1.8	279	4005	8590
5	14.3	-1.7	321	3926	7337
6	8.4	-2.3	331	3944	6605
7	6.6	-2.9	281	4054	9461
8	14.3	-1.7	321	3926	7337
9	13.7	-3.4	329	3905	6534

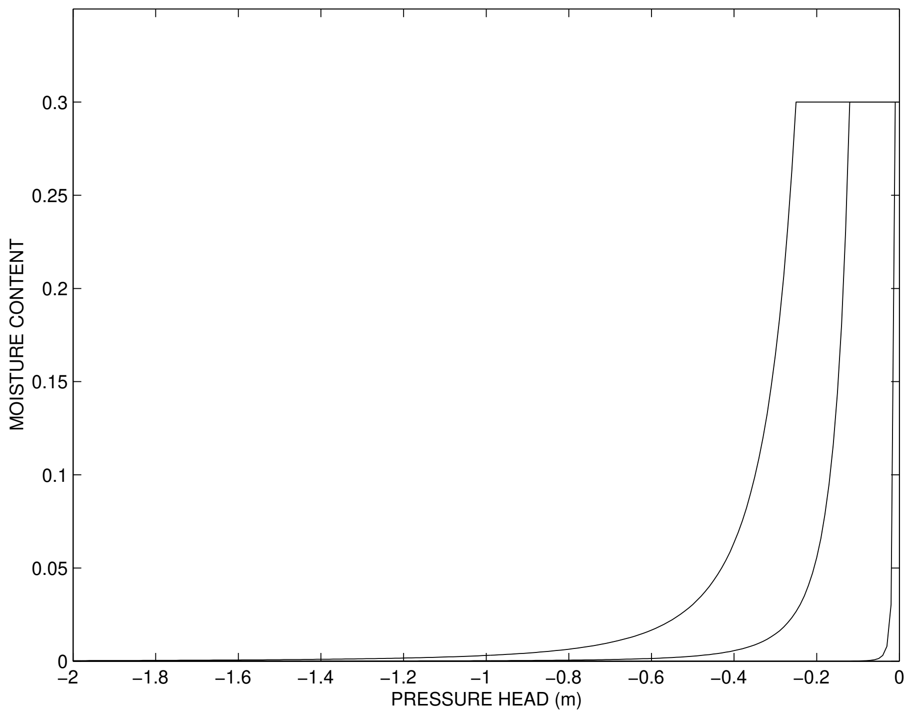

Figure 13: Moisture content as a function of pressure head for (from top to bottom) $\psi_c = -0.25$, -0.12, and -0.01 m.

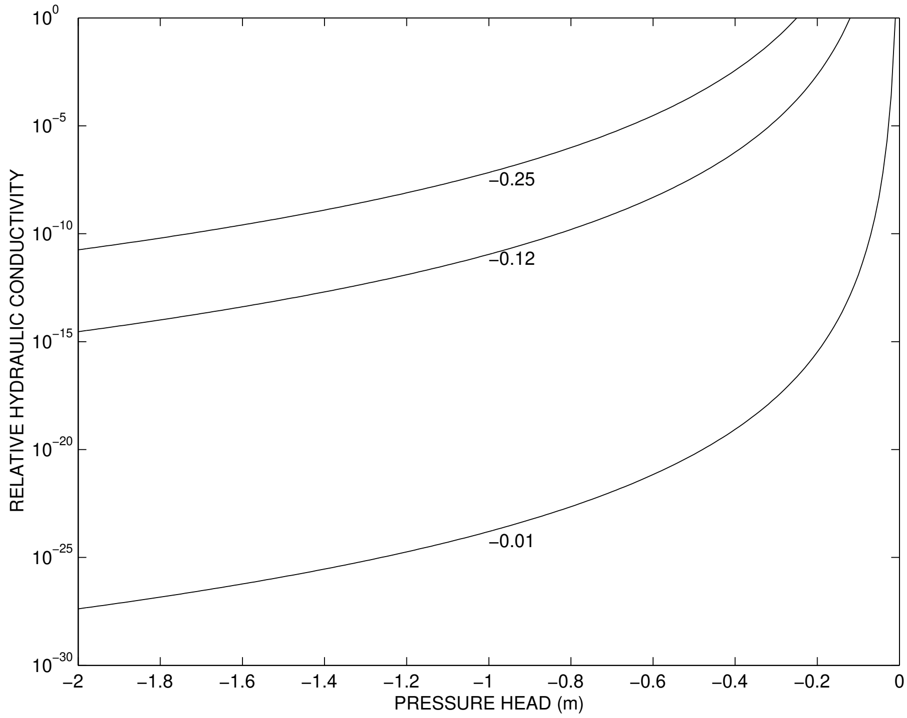

Figure 14: Relative conductivity as a function of pressure head for $\psi_c = -0.25$, -0.12, and -0.01 m.

4 Conclusions

In this study grids were generated for nine characteristic hillslopes in order to conduct simulations for certain test cases with the three-dimensional finite element model CATHY. The results (in the form of outflow hydrographs, water table plots and cumulative flow plots) were displayed and compared to results of the hillslope-storage Boussinesq model. The main results of the study are:

- Hillslopes with a convergent shape show an initial increase in subsurface flow, because of accumulation of water at the bottom of the slope. After about 15 days, the maximum outflow and water table height is reached and both drop gradually. The drainage of these slopes is slow because the outlet is relatively small and forms a 'bottleneck'. In comparison, for straight slopes outflow and water table height decrease gradually from the start, while for divergent slopes outflow and water tables drop rapidly from the start.

- For these simulations errors in mass balance are relatively small: the maximum value is -1.4%.

- Hydrographs and especially water table distributions in time and space obtained with CATHY show a similar pattern to those obtained with the hillslope-storage Boussinesq model. The match is best for convergent hillslopes and is worst for divergent hillslopes. The difference is probably caused by the difference in saturation degree: for high saturation both models are more likely to show similar results.

- Outflow rates obtained with CATHY have higher values than the Boussinesq ones. This is (partially) caused by the fact that the initial conditions for the two models are not exactly the same: the hillslope-storage Boussinesq model has no storage in the unsaturated zone in the initial state whereas the CATHY model does. More significantly, because of the high capillary fringe value used for CATHY simulations, the effect of the unsaturated zone is quite important.

- Using a reduced capillary fringe value, the hydrographs as well as the spatial and temporal distribution of water tables for the CATHY model and the hillslope-storage Boussinesq model show very similar results. As expected the CATHY outflow values decreased. The match between the two models improved and it is expected that for a further decrease of the the ψ_c value the results will be even more alike.

- Mass balance errors for the simulations with a reduced ψ_c value have relatively larger mass balance errors: the maximum value is -3.4%. Furthermore these simulations were more difficult numerically (more CPU, more backstepping, more timesteps) than the original runs. This is due to the increase in nonlinearity and dryness of the soil moisture characteristics as the capillary fringe value is reduced.

From these results we can conclude that:

- Simulation results from CATHY runs are reliable and can be usefully compared to the results of the model based on the Boussinesq equation.

- In order to make the results from both models more comparable, the initial conditions for the two models must be as close as possible. Manipulation of the capillary fringe parameter ψ_c offers this possibility, although with some numerical caveats.

- The importance of unsaturated zone processes on hydrographs and water table distribution is relatively small for the test case that was presented. The model based on the Boussinesq equation is able to capture the dynamic behaviour of hillslope drainage in a way very similar to the CATHY model.

4.1 Future work

Several ideas for future research can be suggested:

- The results from the two models are difficult to compare due to the influence of initial volume of water that is present in the hillslopes. Simulations with initial conditions that are more comparable should be carried out. In particular the influence and handling of the capillary fringe in the CATHY model should be studied more carefully.

- Simulations starting from complete saturation should be carried out in order to determine the drainable porosity values of the slopes more accurately.

- In this study drainage cases for a sandy soil were studied. To get a clearer view of the general influence of the unsaturated zone on hydrographs and water table distributions, simulations for different soil types (e.g., silt, loam) should also be carried out.

- This study examined only subsurface drainage phenomena. Further studies should investigate: rainfall rechare scenarios; alternating rainfall and evaporation episodes; and overland flow generation (in this case running the CATHY model in coupled mode).

- For simulation purposes the original shapes of the nine characteristic hillslopes were changed into a set of hillslopes defined by the location of their divides. The location was determined using a gradient descent or gradient ascent algorithm. The hillslope plan and profile were then straightened, making the resulting hillslope boundaries somewhat arbitrary. This means that the assumption of no flow over the divides of the slope could be in doubt. Moreover this straightening may cause differences between the results obtained with CATHY (which averages water table heights over the width of the hillslope at each point along the hillslope length) and the hillslope-storage Boussinesq model (which calculates one relative storage value for each w-value). These issues should be examined in more detail.

- Intercomparison tests for other hillslope configurations (slope angle, length, soil depth) are needed.

- The CATHY model is DEM-based, but unless a very fine grid is used a DEM discretization (with each DEM cell of the same size) may not accurately capture the flow dynamics for hillslopes with complex shapes (divergence or convergence). On the other hand a very fine DEM discretization may be computationally inefficient, since cells on certain portions of the slope will be much larger than necessary. The discretization for the CATHY model in this study was therefor not DEM-based, but rather it was modified such that the number of cells in the w-direction is kept fixed while the cell length (x-direction) is kept constant, so that the mesh is fine in the places where the hillslope is narrow. These DEM accuracy issues need to be further explored.

- For hillslope configurations that generate surface runoff (e.g., recharge scenarios, or drainage only but on convergent or very steep slopes with higher initial water table position), CATHY should be run to asses different overland routing features (e.g., influence of wave celerity, computational aspects), in comparison also to "instantaneous" routing (uncoupled mode). Hydrologically, an interesting research issue to be investigated here concerns the relative importance of surface routing vs subsurface flow. How is this affected by hillslope angle, length, convergence, etc? How can we quantify these effects? (These investigations will necessitate, however, substantial enhancements to the CATHY model's "hydrograph" post-processing.)

Acknowlegments This work has been supported in part by the European Commission (contract EVK1-CT-2000-00082), by the Italian Ministry of the University (project ISR8, C11-B), and by the Sardinia Regional Authorities.

Appendix: MATLAB pre- and post-processing scripts for the CATHY model

maketinslp3.m: For constructing a triangular network for nine hillslopes

```
function [nxy,nz,ntri,nz_bed] = maketinslp3(type,ndens,soildepth)
%
% This file is used with the  m-file 'run_slopegen3.m'.
%
%
% [nxy,nz,ntri] = maketinslp(type,ndens)
%
% makes a triangular mesh for slopes of the form:
%
% z = E + H*(1-x/L)^n + ay^2
%
% where
% type =      the slope profile shape (cc,cv,cs,sc,sv,ss,vc,vv or vs);
% ndens =     number of elements over slope width
% soildepth = soildepth
%
% Both surfaceTIN and bedrockTIN are generated (bedrock indicated
% with '_bed'). nxy and ntri are the same for surfaceTIN and
% bedrockTIN.
%
% Emiel van Loon - 20/11/2000
% Adapted by A. Hilberts, 16-7-2001

cd ~/matlab/Arno/Data/
     load polyslp_50s;
cd ../
xmax = 100; xmin = 0; x\_bnd = [xmin xmax];

if type == 'cc',L = 105; dz = 5; H = 5.01; n = 2; a = dz/(100^2);
   E = dz-H; ymax= max(divide_cc(:,1)); div = divide_cc; end
if type == 'sc',L = 105; dz = 5; H = 5.25; n = 1; a = dz/(100^2);
   E = dz-H; ymax= max(divide_sc(:,1)); div = divide_sc; end
if type == 'vc',L = 105; dz = 5; H = 8.16; n = 0.31; a = dz/(100^2);
   E = dz-H; ymax= max(divide_vc(:,1)); div = divide_vc; end
if type == 'cs',L = 105; dz = 5; H = 5.01; n = 2; a = 0; E = dz-H;
 ymax= max(divide_cs(:,1)); div = divide_cs; end
if type == 'ss',L = 105; dz = 5; H = 5.25; n = 1; a = 0; E = dz-H;
 ymax= max(divide_ss(:,1)); div = divide_ss; end
if type == 'vs',L = 105; dz = 5; H = 8.16; n = 0.31; a = 0; E = dz-H;
 ymax= max(divide_vs(:,1)); div = divide_vs; end
if type == 'cv',L = 105; dz = 5; H = 5.01; n = 2; a = -dz/(100^2);
   E = dz-H; ymax= max(divide_cv(:,1)); div = divide_cv; end
if type == 'sv',L = 105; dz = 5; H = 5.25; n = 1; a = -dz/(100^2);
   E = dz-H; ymax= max(divide_sv(:,1)); div = divide_sv; end
if type == 'vv',L = 105; dz = 5; H = 8.16; n = 0.31; a = -dz/(100^2);
   E = dz-H; ymax= max(divide_vv(:,1)); div = divide_vv; end


%%%%% for nice plotting, sign should be multy. by +/- 20. %%%%%%
if a==0          % no convergence/divergence
xst = x_bnd(1);xend = x_bnd(2);sign = 1;
end
if a>0         % convergent
xst = x_bnd(1);xend = x_bnd(2);sign = 1;
```

```
xd = x_bnd(2);end
if a<0          % divergent
xst = x_bnd(2);xend = x_bnd(1);sign = -1;
xd = x_bnd(1);end
nodes = [];
x = xst;
xmax = x_bnd(2);

% first create nodes for xst <= x < xend
if a>=0, d = div(1,2); else d = div(end,2); end
while sign*x < sign*xend
   s1 = 2*d./ndens;  ny = [-d:s1:d]';
   nx = x.*ones(ndens+1,1); nodes = [nodes; nx ny];
   x = x + sign*0.5;
   d = interp1(div(:,1),div(:,2),x);
end

% then create the nodes for x = xend
x = xend;
if a>=0, d = div(end,2); else d = div(1,2);end;
s1=2*d./ndens; ny = [-d:s1:d]';
nx = x.*ones(ndens+1,1); nodes = [nodes; nx ny];

% form the final node coordinates
nx = nodes(:,1);
ny = nodes(:,2);
nz = E+H*(1-nx./L).^n + a*ny.^2;

% now form triangules, strip by strip:
ntri = [];
nrstrip = length(nx)./(ndens+1);
for i=1:nrstrip-1
  start = (i-1)*(ndens+1)+1;
  stop = (i+1)*(ndens+1);
  xtri = nx(start:stop);
  ytri = ny(start:stop);
  tri = delaunay(xtri,ytri);
  ntri = [ntri;tri+start-1];
end


%%%%%%%%%%%%%%% Correction of 16-7-2001 %%%%%%%%%%%%%%
nz = Es + Hs*(1-nx./L).^1 + 0;% straight plan and prof
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% output
nxy =[nx ny]; nz_bed = nz - soildepth;
```

run_maketinslp3.m: For running "maketinslp3"

```
% 'run_maketinslp3.m' runs the file 'maketinslp3.m' and saves plots and\\
% generated data to disk. A value for 'ndens' has to be filled in. See \\
% 'maketinslp3.m' for more information.\\
%\\
% Written by A. Hilberts, 16-07-2001.\\

clear all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%% FILL IN VARIABLE 'ndens' ans 'soildepth' %%%%%%%%%%%%%

ndens = 6;
soildepth = 2;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%polyslp_50s%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
[nxy_cc,nz_cc,ntri_cc,nz_bed_cc] = maketinslp3('cc',ndens,soildepth);
[nxy_cv,nz_cv,ntri_cv,nz_bed_cv] = maketinslp3('cv',ndens,soildepth);
[nxy_cs,nz_cs,ntri_cs,nz_bed_cs] = maketinslp3('cs',ndens,soildepth);
[nxy_sc,nz_sc,ntri_sc,nz_bed_sc] = maketinslp3('sc',ndens,soildepth);
[nxy_sv,nz_sv,ntri_sv,nz_bed_sv] = maketinslp3('sv',ndens,soildepth);
[nxy_ss,nz_ss,ntri_ss,nz_bed_ss] = maketinslp3('ss',ndens,soildepth);
[nxy_vc,nz_vc,ntri_vc,nz_bed_vc] = maketinslp3('vc',ndens,soildepth);
[nxy_vv,nz_vv,ntri_vv,nz_bed_vv] = maketinslp3('vv',ndens,soildepth);
[nxy_vs,nz_vs,ntri_vs,nz_bed_vs] = maketinslp3('vs',ndens,soildepth);
% number of 2D-nodes and triangles;
[numberofnodes_cc,fake] = size(nxy_cc);numberoftri_cc = size(ntri_cc);
[numberofnodes_cv,fake] = size(nxy_cv);numberoftri_cv = size(ntri_cv);
[numberofnodes_cs,fake] = size(nxy_cs);numberoftri_cs = size(ntri_cs);
[numberofnodes_vc,fake] = size(nxy_vc);numberoftri_vc = size(ntri_vc);
[numberofnodes_vv,fake] = size(nxy_vv);numberoftri_vv = size(ntri_vv);
[numberofnodes_vs,fake] = size(nxy_vs);numberoftri_vs = size(ntri_vs);
[numberofnodes_sc,fake] = size(nxy_sc);numberoftri_sc = size(ntri_sc);
[numberofnodes_sv,fake] = size(nxy_sv);numberoftri_sv = size(ntri_sv);
[numberofnodes_ss,fake] = size(nxy_ss);numberoftri_ss = size(ntri_ss);
cd ~/matlab/Arno/Data/
    save polyslp_50s_xyztri3 *;
cd ../
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% testplot
nx_cc=nxy_cc(:,1);ny_cc=nxy_cc(:,2);nx_cv=nxy_cv(:,1);ny_cv=nxy_cv(:,2);
nx_cs=nxy_cs(:,1);ny_cs=nxy_cs(:,2);nx_sc=nxy_sc(:,1);ny_sc=nxy_sc(:,2);
nx_sv=nxy_sv(:,1);ny_sv=nxy_sv(:,2);nx_ss=nxy_ss(:,1);ny_ss=nxy_ss(:,2);
nx_vc=nxy_vc(:,1);ny_vc=nxy_vc(:,2);nx_vv=nxy_vv(:,1);ny_vv=nxy_vv(:,2);
nx_vs=nxy_vs(:,1);ny_vs=nxy_vs(:,2);

subplot(3,3,1), trimesh(ntri_cc,nx_cc,ny_cc,nz_cc);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]); title('1')
subplot(3,3,2), trimesh(ntri_cs,nx_cs,ny_cs,nz_cs);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('2')
subplot(3,3,3), trimesh(ntri_cv,nx_cv,ny_cv,nz_cv);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('3')
subplot(3,3,4), trimesh(ntri_sc,nx_sc,ny_sc,nz_sc);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('4')
subplot(3,3,5), trimesh(ntri_ss,nx_ss,ny_ss,nz_ss);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('5')
subplot(3,3,6), trimesh(ntri_sv,nx_sv,ny_sv,nz_sv);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('6')
subplot(3,3,7), trimesh(ntri_vc,nx_vc,ny_vc,nz_vc);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('7')
```

```
subplot(3,3,8), trimesh(ntri_vs,nx_vs,ny_vs,nz_vs);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('8')
subplot(3,3,9), trimesh(ntri_vv,nx_vv,ny_vv,nz_vv);
set(gca,'CameraPosition',[-1100 450 -60],'YTicklabel',[])
axis([0 100 -25 25 0 10]);title('9')
cd ~/figures/
     print -deps polyslp_50s_TIN3
cd ~/matlab/Arno
```

watersurf2.m: For calculating and plotting water tables and hydrographs

```
% The function watersurf computes watertables for the whole x,y-domain of
% the 9 characteristic hillslope from inputfiles xyz and psi
% (generated by 'cathy').
%
% Version 13-6-2001, written by A. Hilberts and Rijk Oosterhof, CRS4 /
%                                         Wageningen University
%
% Input variables:
%               stype =     slopetype ('cc', 'cs', 'cv', and so on)
%               soildepth=  soildepth of hillslopes     (m)
%
% Outputvariables:
%           wt =       watertable matrix. The columnvectors contain
%                      watertable values for all (surface)nodes.
%                      A value of 0.85 for wt(3,2) indicates that for
%                      the second node at the third timeprint the water-
%                      table is 0.85 meters above the bedrock.
%           wt_centroid =
%                      matrix of watertablevalues only for the centroid
%                      profile (the middle nodes of the hillslope).
%
% When the function 'watersurf.m' is called, it saves the outputvariables
% and the corresponding time-vector (TIME) in a file called 'watertable.mat'.
% It also prints several figures (to file) in -eps or -ps format.
% Both the -eps / -ps files  and the -mat file are written to the
% directories
% /nfsdata/environ/cathy_uncoupled/i-o/hillslope/arno/figures_50s_**  in which
% '**' stands for hillslopetype (e.g. 'cc', 'cs', ...,'vv')
%
% Besides these outputvariables, several plots are saved when calling this
% function.
%  1) In the file 'plot_hg2.m' hydrographs and cumalative flowvolume (!)
%     are plotted.
%  2) In the file 'plot_watertable2.m' the watertables (wt and wt_centroid)
%     are calculated, plotted and saved.
%
% Assumption is that over the width of the slope there are 7 nodes,
% in the length direction there are 201 nodes and the slopes have dimension
% 50 x 100 m.

function [wt,wt_centroid]=watersurf2(stype,soildepth)

%%%%%%%%%%%%%% HANDLES FOR SHORTAGE OF INPUT %%%%%%%%%%%%%%%%%%%%
if nargin < 2, soildepth = 2; end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if stype=='cs', disp('Slopetype is cs');
```

```
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_*s/,
end
if stype=='ss', disp('Slopetype is ss');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_*s/
end
if stype=='vs', disp('Slopetype is vs');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_*s/
end
if stype=='sc', disp('Slopetype is sc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_sc/
end
if stype=='sv', disp('Slopetype is sv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_sv/
end
if stype=='cc', disp('Slopetype is cc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_cc/
end
if stype=='cv', disp('Slopetype is cv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_cv/
end
if stype=='vc', disp('Slopetype is vc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_vc/
end
if stype=='vv', disp('Slopetype is vv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/output_vv/
end
%%%%%%%%%%%%%%%%%% LOAD INPUT FILES (set 1)%%%%%%%%%%%%%%%%%%%%%%
load tri; load xy; load z; tri=tri(:,1:3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

wt=[]; wt_centroid=[]; XYZsurface=[]; Z=[];
a=0; b=0; c=0; minx=0; miny=0; maxx=0; maxy=0; nstr=0;
% (nstr= number of layers)

%%%%%%%%%%%%%%%%%% READ INPUT FILES (set 2) %%%%%%%%%%%%%%%%%%%%%%
disp('Reading datafiles');
xyz = fopen('xyz','r');
psi = fopen('psi','r');
nnod = fscanf(xyz,'%u %u',2);       % Read header
nstr = nnod(2)/nnod(1);    % nnod=[(# of surface nodes) (# of nodes)]
fgets(xyz);                         % Skip line
XYZ = fscanf(xyz,'%g',[4,nnod(2)]); % Read data
XYZ = XYZ';
XYZsurface=XYZ(1:nnod(1),:);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd ~/matlab/Arno/

for a=0:nstr-1            % Create Z, contains elevation of each node.
    b=a+1;                % Vertical profiles in rows, so that the lowest
    for c=1:nnod(1)       % nodes are in column 1.
        Z(c,b)=XYZ(a*nnod(1)+c,4);
    end
end
Z = fliplr(Z);
fclose(xyz);

maxx=max(XYZsurface(:,2)); maxy=max(XYZsurface(:,3));
minx=min(XYZsurface(:,2)); miny=min(XYZsurface(:,3));
b=0; c=0; d=0; nprt=0; % nprt = number of time values for detailed nodal output
rc=0; z0=0; i=[]; TIME=[]; PSI=[]; VPPSI=[]; Z0=[];FILENAME=[]; FLAG=[];
```

```
% FLAG, flag to indicate certain occurances
% 1 watertable calculated correctly
% 2 unstaturated zone above saturated zone above unsaturated zone
% 3 watertable not encountered, fully saturated vertical profile
% 4 watertable not encountered, unsaturated profile

while (b~=(-1))         % Find out number of time values for
                        % detailed nodal output (nprt)
      b = fgetl(psi);
      if (b~=(-1))      % Loops trough psi until b=-1 end of file
      c = double(b);
      end
      if size(c)<33, c(30:33)=0; end
      if c(29:33)==[78 83 84 69 80] % c(29:33)='NSTEP'
         nprt = nprt+1;
      end
end
frewind(psi); FLAG = zeros(nnod(1),nprt);

%%%%%%%%%%% LOCATION OF WATERTABLE AND PLOTTING %%%%%%%%%%%%%%%%%%%%%%%%
disp('Locating watertable');
for a=1:nprt
    % Z0 contains for every vertical profile the height of the watertable,
    % default is the watertable at highest node
    Z0 = XYZsurface(:,4);
    time = fscanf(psi,'%u %g',[1,2]);
    TIME = [TIME;time]; % TIME contains timestep and times of nprt
    fgets(psi);
    % Read in data from psi
    PSI = fscanf(psi,'%g',[(nstr*nnod(1)),1]);
    for b=0:nstr-1        % Create VPPSI, contains psi of each node.
        c=b+1;            % Vertical profiles in rows and layers in columns,
        for d=1:nnod(1)   % so that the bottom layer is in column 1.
            VPPSI(d,c)=PSI(b*nnod(1)+d);
        end
    end
    VPPSI = fliplr(VPPSI);
    % Start from the bottom of the profile, and proceed upwards
    % until node with negative pressure head is encountered.
    % Calculate watertable between this node and the one directly below
    % by lineair interpolation.
    for b=1:nnod(1)
        for c=1:nstr-1
            if VPPSI(b,c)>0 & VPPSI(b,c+1)<0 & FLAG(b,a)==0
               % Watertable, interpolate linearly (Z=rc*VPPSI+z0)
               rc = (Z(b,c)-Z(b,c+1))/(VPPSI(b,c)-VPPSI(b,c+1));
               Z0(b) = Z(b,c)-rc*VPPSI(b,c);
               FLAG(b,a)=1;
               elseif VPPSI(b,c)>0 & VPPSI(b,c+1)<0 & FLAG(b,a)==1
               % Second watertable
               FLAG(b,a)=2;
               elseif c==nstr-1 & VPPSI(b,c+1)>=0 & FLAG(b,a)==0
               % Watertable not encounterd and nodes stil saturated
               FLAG(b,a)=3;
               elseif c==nstr-1 & FLAG(b,a)==0
               % Watertable not encounterd, and not fully saturated
               % profile. Set watertable to lowest node
               Z0(b) = Z(b,c-nstr+2);
               FLAG(b,a)=4;
               end
        end
```

```
    end
    %%%%% This Z0 contains values in m above bedrock %%%%%%%%%%%%%
    % Z0 = XYZsurface(:,4)-Z0;
    Z0 = Z0 - (XYZsurface(:,4) - soildepth);
    %%% construct wt matrix with z0 for the different timesteps %%%
    wt=[wt, Z0];

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    disp('Plotting...')

    %%%%%%%%%%%%%%% PLOTTING %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    figure(a); trisurf(tri,xy(:,1),xy(:,2),Z0); hold on
    grid off, shading flat, colormap (1-gray)
    % draw edges
    x=[0:10:100]; y=25*ones(11,1); plot(x,y,'k-');
    y=-25*ones(11,1); plot(x,y,'k-');
    x=[0 0]; y= [-25 25]; plot(x,y,'k-');
    x=[100 100]; y= [-25 25]; plot(x,y,'k-');
    title(['Water table distribution (m above bedrock) at ',...
num2str(round(TIME(a,2)/86400)),' days'])
    xlabel('(m)'), ylabel('(m)'), hold off,
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%% ADAPT AND SAVE PLOTS %%%%%%%%%%%%%%%%%%%%%%%
if stype=='cs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s/, end
if stype=='ss', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s/, end
if stype=='vs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s/, end
if stype=='sc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sc/, end
if stype=='sv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sv/, end
if stype=='cc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cc/, end
if stype=='cv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cv/, end
if stype=='vc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vc/, end
if stype=='vv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vv/, end

for a=1:nprt
    figure(a)
    caxis ([0 1.5]), colorbar, axis([0 100 -30 30 0 1.5]);
    % Create postscript outputfile
    FILENAME = ['graphwatertabled',num2str(round(TIME(a,2)/86400))];
    print('-deps','-r200',FILENAME);
end
TIME=TIME/86400; TIME=round(TIME);
FLAG(:,nprt+1) = XYZsurface(:,1);
wt_centroid = wt(4:7:nnod,:); fclose(psi);
wt=double(wt); wt_centroid = double(wt_centroid);
save watertable.mat wt wt_centroid TIME
%%%%%%%%% Recalculation of watertables, after straightening %%%%%%%%%
cd ~/temp
save watertable.mat wt wt_centroid TIME
```

```
cd ~/matlab/Arno
[dummy]=recalcwatertable(stype);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% RUN SCRIPT-FILES 'plot_watertable2.m','plot_hg2.m' and %%%%%%%%%%%%%%
% 'plot_surfhg1.m'. %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all; plot_watertable2;
close all; plot_hg2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%clear all;
```

plot_hg2.m: For reading and plotting of hydrographs and cumulative flow

```
% File for reading the cathy-output files and plotting hydrographs.
% The file is called by 'watersurf2.m' but can also be called individually.
%
% Written by A. Hilberts, 17-5-2001, CRS4 / Wageningen University
%

%clear all;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%% LOAD DATA %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if isempty(stype)==1;
    stype=input('Choose slopetype (e.g. "cc", "cs", "cv" and so on....)         ');
else
    stype=stype;
end
if stype=='cs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='ss', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='vs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='sc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sc, end
if stype=='sv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sv, end
if stype=='cc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cc, end
if stype=='cv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cv, end
if stype=='vc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vc, end
if stype=='vv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vv, end

%%%%%%%%%%%%%%%%%%%%%%%%%%%% LOAD DATA %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
load watertable.mat;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if stype=='cs', cd ../output_*s/, end
if stype=='ss', cd ../output_*s/, end
if stype=='vs', cd ../output_*s/, end
if stype=='sc', cd ../output_sc/, end
if stype=='sv', cd ../output_sv/, end
if stype=='cc', cd ../output_cc/, end
if stype=='cv', cd ../output_cv/, end
if stype=='vc', cd ../output_vc/, end
if stype=='vv', cd ../output_vv/, end
load hgnansfdirdet; load cumflowvol;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%% INPUT DATA (meters, seconds)%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
timevp=TIME(:,2);  %%%%%% time of verticale profile output (meters, days)
data=hgnansfdirdet;
time = data(:,3); flow = -data(:,5);
cumflowvol = -cumflowvol(:,7);
%%%%%%%%%%%% CHANGE UNITS TO: meters, days %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
time = time/86400; flow = flow*86400;

%%%%%%%%%%%%%%% Plot hydrograph(s) and cumflowvol %%%%%%%%%%%%%%%%%%%%%%
if stype=='cs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='ss', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='vs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='sc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sc, end
if stype=='sv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sv, end
if stype=='cc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cc, end
if stype=='cv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cv, end
if stype=='vc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vc, end
if stype=='vv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vv, end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PLOTS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1); plot(time,flow,'k'); xlabel('TIME (days)');
ylabel('FLOW (m^3/day)');
figure(2); plot(time,flow,'k'); xlabel('TIME (days)');
ylabel('FLOW (m^3/day)');
axis([0 1.2e+5 0 1e-2]); %%%%%%%% zoom-in on tail of hg
figure(3); plot(time,flow,'k'); xlabel('TIME (days)');
ylabel('FLOW (m^3/day)');
axis([0 .02 0 1e+1]);    %%%%%%%% zoom-in on head of hg
figure(4); plot(time,cumflowvol,'k');
xlabel('TIME (days)'); ylabel('CUMALATIVE FLOW (m^3)');
%%% To get more detail on the cumflowplot, zoom in up to the point
%%% where cumflowvol <= 0.90 * cumflowvol(end)
multfc = cumflowvol<0.9*cumflowvol(end); cumfl_zoom = multfc.*cumflowvol;
nz = nnz(cumfl_zoom); %%%%%%%% determine # of non-zero elements
figure(5); plot(time(1:nz),cumfl_zoom(1:nz),'k') % plot non-zero elements
xlabel('TIME (days)'); ylabel('CUMALATIVE FLOW (m^3)');

save flowdata time flow cumflowvol

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% PRINTING %%%%%%%%%%%%%%%%%%%%%%%%
figure(1); print -deps hg;
figure(2); print -deps hg_tail;
figure(3); print -deps hg_head;
figure(4); print -deps cumflowvol;
figure(5); print -deps cumflowvol_head;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd ~/temp
save flowdata time flow cumflowvol
cd ~/matlab/Arno/
```

plot_watertable2: For reading and plotting of water tables

```
% This script loads the file 'watertable.mat' (which can be generated
% using 'watersurf.m' and makes plots of the watertable at different times.
% The scriptfile is called by 'watersurf2.m' but can also be called individually.
%
% Written by A. Hilberts, 24-5-2001, CRS4 / Wageningen University

%clear all;
%close all;
if isempty(stype)==1;
    stype=input('Choose slopetype (e.g. "cc", "cs", "cv" and so on...)      ');
else
    stype=stype;
end

if stype=='cs',disp('Slopetype is cs');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
if stype=='ss',disp('Slopetype is ss');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
if stype=='vs',disp('Slopetype is vs');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
if stype=='sc',disp('Slopetype is sc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_sc/,
    x=[0:.5:100];
end
if stype=='sv',disp('Slopetype is sv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_sv/,
    x=[100:-.5:0];
end
if stype=='cc', disp('Slopetype is cc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_cc/,
    x=[0:.5:100];
end
if stype=='cv', disp('Slopetype is cv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_cv/,
    x=[100:-.5:0];
end
if stype=='vc',disp('Slopetype is vc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_vc/,
    x=[0:.5:100];
end
if stype=='vv',disp('Slopetype is vv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_vv/,
    x=[100:-.5:0];
end

%%%%% Load input %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
load watertable.mat; time=TIME(:,2); load xy;
y_width = xy(701:707,2);

%%%%%%%%%%%%PLOT%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1), plot(x,wt_centroid,'k');
title('Watertable (m above bedrock) along centroid profile at different timesteps');
xlabel('SLOPELENGTH (m)'); ylabel('h (m)');
```

```
figure(2), plot(y_width,wt(701:707,:),'k');
axis([y_width(1) y_width(end) 0 1.1]);
title('Watertable (m above bedrock) perpendicular to centroid profile at x = 50 m ...
at different timesteps');
xlabel('SLOPEWIDTH (m)'); ylabel('h (m)');


if stype=='cs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='ss', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='vs', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_*s, end
if stype=='sc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sc, end
if stype=='sv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_sv, end
if stype=='cc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cc, end
if stype=='cv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_cv, end
if stype=='vc', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vc, end
if stype=='vv', cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/...
figures_50s_vv, end
figure(1); print -dps watertable_length;
figure(2); print -dps watertable_width;
cd ~/temp
save watertable.mat wt wt_centroid TIME
cd ~/matlab/Arno/
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

recalcwatertable.m: For calculating a width-averaged water table height

```
% Function to recalculate watertables, since
% the watertables in the plan-direction have a
% curved shape.
%
% A. Hilberts, 24-7-2001, CRS4 / Wageningen Universiteit
%
%
function [dummy]=recalcwatertable(stype)
%
%

if isempty(stype)==1;
    stype=input('Choose slopetype (e.g. "cc", "cs", "cv" and so on...)      ');
else
    stype=stype;
end

if stype=='cs',disp('Slopetype is cs');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
if stype=='ss',disp('Slopetype is ss');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
```

```
if stype=='vs',disp('Slopetype is vs');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_*s/,
    x=[0:.5:100];
end
if stype=='sc',disp('Slopetype is sc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_sc/,
    x=[0:.5:100];
end
if stype=='sv',disp('Slopetype is sv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_sv/,
    x=[100:-.5:0];
end
if stype=='cc', disp('Slopetype is cc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_cc/,
    x=[0:.5:100];
end
if stype=='cv', disp('Slopetype is cv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_cv/,
    x=[100:-.5:0];
end
if stype=='vc',disp('Slopetype is vc');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_vc/,
    x=[0:.5:100];
end
if stype=='vv',disp('Slopetype is vv');
    cd /nfsdata/environ/arno/cathy_uncoupled/i-o/hillslope/figures_50s_vv/,
    x=[100:-.5:0];
end

load watertable.mat
wt_centroid=[];
for j=1:10
    for i=0:200
        wt_centroid(i+1,j)=mean(wt(((7*i+1):(7*i+7)),j));
    end
end
save watertable wt wt_centroid TIME;
clear all;
dummy=[];
cd ~/matlab/Arno
```

subplot_watertable2_1.m: Subplotting and saving relative storage for nine hillslopes

```
% This scriptfile makes subplots of the watertable
%
% Written by A. Hilberts, CRS4 / Wageningen University
% Cagliari, 21-06-2001
%
% Note: This file needs to be run to generate the file
% 'watertables.mat' that is loaded in 'compare1.m' in order
% to make plots of both Cathy and Boussinesq wt's.
%
%%%%%%%%%%%%%%%%%%% MAKE SUBPLOTS OF  WATERTABLE%%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
close all;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%% FILL IN SOILDEPTH %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
soildepth=2;
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

stype='cc'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,1); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('1');
wt_cc=wt_centroid(:,(1:7))/soildepth;x_cc=x;
save watertables wt_cc x_cc
text(5,0.2,'0');text(5,0.1,'2');text(15,0.1,'5');text(25,0.1,'10');
text(45,0.1,'20');text(85,0.1,'50');


stype='cs'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,2); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('2');
wt_alls=wt_centroid(:,(1:7))/soildepth;x_alls=x;
save watertables wt_alls x_alls -append


stype='cv'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,3); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('3');
wt_cv=wt_centroid(:,(1:7))/soildepth;x_cv=x;
save watertables wt_cv x_cv -append


stype='sc'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,4); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('4');
wt_sc=wt_centroid(:,(1:7))/soildepth;x_sc=x;
save watertables wt_sc x_sc -append

stype='ss'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,5); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('5');

stype='sv'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,6); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('6');
wt_sv=wt_centroid(:,(1:7))/soildepth;x_sv=x;
save watertables wt_sv x_sv -append

stype='vc'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,7); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('7');xlabel('DISTANCE (m)');
ylabel('RELATIVE STORAGE (m/m)');
wt_vc=wt_centroid(:,(1:7))/soildepth;x_vc=x;
save watertables wt_vc x_vc -append

stype='vs'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,8); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('8');

stype='vv'; plot_watertable2; figure(3);wt_centroid(:,2)=zeros(201,1);
subplot(3,3,9); plot(x,(wt_centroid(:,(1:7))/soildepth),'k');
axis([0 100 0 0.6]);title('9');
wt_vv=wt_centroid(:,(1:7))/soildepth;x_vv=x;
save watertables wt_vv x_vv -append


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SAVE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd /nfsdata/environ/arno/figures/
```

```
figure(3), print -deps subplot_rS;
figure(3), print subplot_rS;
cd ~/matlab/Arno/
```

subplot_hg2.m: Subplotting and saving hydrographs

```
% This script runs plot_hg2.m and makes subplots of
% different types of hydrographs.
%
% Note: This file needs to be run to generate the file
% 'flows.mat' that is loaded in 'compare1.m' in order
% to make plots of both Cathy and Boussinesq hg's.
%
% Written by A. Hilberts, 25-6-2001, CRS4 / Wageningen Univerity

cd ~/matlab/Arno/
clear all; close all;
area=[2495.7 5000 646.2277 2160.2 5000 2160.9 1409.6 5000 2385.8];

stype='cc'; plot_hg2; figure(10);
flow=flow*1000/area(1);
subplot(3,3,1); plot(time,flow,'k');
axis([0 50 0 20]);title('1');cd ~/matlab/Arno;
flow_cc=flow;time_cc=time;
save flows flow_cc time_cc;

stype='cs'; plot_hg2; figure(10);
flow=flow*1000/area(2);
subplot(3,3,2); plot(time,flow,'k');
axis([0 50 0 20]);title('2');cd ~/matlab/Arno;
flow_alls=flow;time_alls=time;
save flows flow_alls time_alls -append;

stype='cv'; plot_hg2; figure(10);
flow=flow*1000/area(3);
subplot(3,3,3); plot(time,flow,'k');
axis([0 50 0 20]);title('3');cd ~/matlab/Arno;
flow_cv=flow;time_cv=time;
save flows flow_cv time_cv -append;

stype='sc'; plot_hg2; figure(10);
flow=flow*1000/area(4);
subplot(3,3,4); plot(time,flow,'k')
axis([0 50 0 20]);title('4');cd ~/matlab/Arno;
flow_sc=flow;time_sc=time;
save flows flow_sc time_sc -append ;

stype='ss'; plot_hg2; figure(10);
flow=flow*1000/area(5);
subplot(3,3,5); plot(time,flow,'k');
axis([0 50 0 20]);title('5');cd ~/matlab/Arno;

stype='sv'; plot_hg2; figure(10);
flow=flow*1000/area(6);
subplot(3,3,6); plot(time,flow,'k');
axis([0 50 0 20]);title('6');cd ~/matlab/Arno;
flow_sv=flow;time_sv=time;
save flows flow_sv time_sv -append;
```

```
stype='vc'; plot_hg2; figure(10);
flow=flow*1000/area(7);
subplot(3,3,7); plot(time,flow,'k');
axis([0 50 0 20]);title('7');cd ~/matlab/Arno;
xlabel('TIME (days)'); ylabel('SUBSURFACE FLOW (mm/day)');
flow_vc=flow;time_vc=time;
save flows flow_vc time_vc -append;

stype='vs'; plot_hg2; figure(10);
flow=flow*1000/area(8);
subplot(3,3,8); plot(time,flow,'k');
axis([0 50 0 20]);title('8');cd ~/matlab/Arno;

type='vv'; plot_hg2; figure(10);
flow=flow*1000/area(9);
subplot(3,3,9); plot(time,flow,'k');
axis([0 50 0 20]);title('9');cd ~/matlab/Arno;
flow_vv=flow;time_vv=time;
save flows flow_vv time_vv -append;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SAVE %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd /nfsdata/environ/arno/figures/
figure(10), print -deps subplot_rQ;
cd ~/matlab/Arno/
```

compare1.m: For comparing CATHY results to the hillslope-storage Boussinesq model

```
% File to compare Boussinesq results with the CATHY results
%
% Written by A. Hilberts, 1-8-2001
% CRS4 / Wageningen University
%
%
% The .mat files 'flows.mat' and 'watertables.mat' are called to
% compare the results. 'flows.mat' is generated while running
% the file 'subplot_hg2.m'. The file 'watertables.mat' is
% generated in 'subplot_watertable2_1.m'.
% 'flows.mat' contains flow_** and time_** values (where **
% stands for hillslope type) and 'watertables.mat' contains
% wt_** and x_** values.
% Note: the files that generate flows.mat and watertables.mat
% save the .mat files in directory ~/matlab/Arno and they are
% called from ~/results: they have to be copied first.
clear all;
close all;
%%%%%%%%%%%%%%%%%%%%% LOAD DATA %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd ~/figures
load out_0_0_polyslp_50s
cd ~/results
load flows
load watertables
area=[2495.7 5000 646.2277 2160.2 5000 2160.9 1409.6 5000 2385.8];
%%%%%%%%%%%%%%%%%%%%%% CHANGE UNITS %%%%%%%%%%%%%%%%%%%%%%%%%
Q_cc = Q_cc_fvn*86400*1000/area(1);
Q_cs = Q_cs_fvn*86400*1000/area(2);
Q_cv = Q_cv_fvn*86400*1000/area(3);
Q_sc = Q_sc_fvn*86400*1000/area(4);
```

```
Q_ss = Q_ss_fvn*86400*1000/area(5);
Q_sv = Q_sv_fvn*86400*1000/area(6);
Q_vc = Q_vc_fvn*86400*1000/area(7);
Q_vs = Q_vs_fvn*86400*1000/area(8);
Q_vv = Q_vv_fvn*86400*1000/area(9);

H_cc = H_cc_fvn/2;
H_cc2 = H_cc(3,:); H_cc5 = H_cc(6,:); H_cc10 = H_cc(11,:);
H_cc20 = H_cc(21,:); H_cc50 = H_cc(51,:);
H_cc=[H_cc2;H_cc5;H_cc10;H_cc20;H_cc50];

H_cs = H_cs_fvn/2;
H_cs2 = H_cs(3,:); H_cs5 = H_cs(6,:); H_cs10 = H_cs(11,:);
H_cs20 = H_cs(21,:); H_cs50 = H_cs(51,:);
H_cs=[H_cs2;H_cs5;H_cs10;H_cs20;H_cs50];

H_cv = H_cv_fvn/2;
H_cv2 = H_cv(3,:); H_cv5 = H_cv(6,:); H_cv10 = H_cv(11,:);
H_cv20 = H_cv(21,:); H_cv50 = H_cv(51,:);
H_cv=[H_cv2;H_cv5;H_cv10;H_cv20;H_cv50];

H_sc = H_sc_fvn/2;
H_sc2 = H_sc(3,:); H_sc5 = H_sc(6,:); H_sc10 = H_sc(11,:);
H_sc20 = H_sc(21,:); H_sc50 = H_sc(51,:);
H_sc=[H_sc2;H_sc5;H_sc10;H_sc20;H_sc50];

H_ss = H_ss_fvn/2;
H_ss2 = H_ss(3,:); H_ss5 = H_ss(6,:); H_ss10 = H_ss(11,:);
H_ss20 = H_ss(21,:); H_ss50 = H_ss(51,:);
H_ss=[H_ss2;H_ss5;H_ss10;H_ss20;H_ss50];

H_sv = H_sv_fvn/2;
H_sv2 = H_sv(3,:); H_sv5 = H_sv(6,:); H_sv10 = H_sv(11,:);
H_sv20 = H_sv(21,:); H_sv50 = H_sv(51,:);
H_sv=[H_sv2;H_sv5;H_sv10;H_sv20;H_sv50];

H_vc = H_vc_fvn/2;
H_vc2 = H_vc(3,:); H_vc5 = H_vc(6,:); H_vc10 = H_vc(11,:);
H_vc20 = H_vc(21,:); H_vc50 = H_vc(51,:);
H_vc=[H_vc2;H_vc5;H_vc10;H_vc20;H_vc50];

H_vs = H_vs_fvn/2;
H_vs2 = H_vs(3,:); H_vs5 = H_vs(6,:); H_vs10 = H_vs(11,:);
H_vs20 = H_vs(21,:); H_vs50 = H_vs(51,:);
H_vs=[H_vs2;H_vs5;H_vs10;H_vs20;H_vs50];

H_vv = H_vv_fvn/2;
H_vv2 = H_vv(3,:); H_vv5 = H_vv(6,:); H_vv10 = H_vv(11,:);
H_vv20 = H_vv(21,:); H_vv50 = H_vv(51,:);
H_vv=[H_vv2;H_vv5;H_vv10;H_vv20;H_vv50];

t=t/24;
x_bouss=[100:-1:0];
% flows.mat contains flow_** and time_**

%%%%%%%%%%%%%%%%PLOTTING HG's %%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(1);
subplot(3,3,1);plot(time_cc,flow_cc,'k');axis([0 50 0 20]);
hold on; plot(t,Q_cc,'k--');title('1');

subplot(3,3,2);plot(time_alls,flow_alls,'k');axis([0 50 0 20]);
```

```
hold on; plot(t,Q_cs,'k--');title('2');

subplot(3,3,3);plot(time_cv,flow_cv,'k');axis([0 50 0 20]);
hold on; plot(t,Q_cv,'k--');title('3');

subplot(3,3,4);plot(time_sc,flow_sc,'k');axis([0 50 0 20]);
hold on; plot(t,Q_sc,'k--');title('4');

subplot(3,3,5);plot(time_alls,flow_alls,'k');axis([0 50 0 20]);
hold on; plot(t,Q_ss,'k--');title('5');

subplot(3,3,6);plot(time_sv,flow_sv,'k');axis([0 50 0 20]);
hold on; plot(t,Q_sv,'k--');title('6');

subplot(3,3,7);plot(time_vc,flow_vc,'k');axis([0 50 0 20]);
hold on; plot(t,Q_vc,'k--');title('7');
xlabel('TIME (days)');ylabel('SUBSURFACE FLOW (mm/day)');
subplot(3,3,8);plot(time_alls,flow_alls,'k');axis([0 50 0 20]);
hold on; plot(t,Q_vs,'k--');title('8');

subplot(3,3,9);plot(time_vv,flow_vv,'k');axis([0 50 0 20]);
hold on; plot(t,Q_vv,'k--');title('9');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%% PLOTTING WT's %%%%%%%%%%%%%%%%%%%%%%%%%%%%
figure(2);
subplot(3,3,1);plot(x_cc,wt_cc,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_cc,'k--');title('1');

subplot(3,3,2);plot(x_alls,wt_alls,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_cs,'k--');title('2');

subplot(3,3,3);plot(x_cv,wt_cv,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_cv,'k--');title('3');

subplot(3,3,4);plot(x_sc,wt_sc,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_sc,'k--');title('4');

subplot(3,3,5);plot(x_alls,wt_alls,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_ss,'k--');title('5');

subplot(3,3,6);plot(x_sv,wt_sv,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_sv,'k--');title('6');

subplot(3,3,7);plot(x_vc,wt_vc,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_vc,'k--');title('7');
xlabel('DISTANCE (m)');ylabel('RELATIVE STORAGE (m/m)');

subplot(3,3,8);plot(x_alls,wt_alls,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_vs,'k--');title('8');

subplot(3,3,9);plot(x_vv,wt_vv,'k');axis([0 100 0 0.6]);
hold on; plot(x_bouss,H_vv,'k--');title('9');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%% PRINT PLOTS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
cd /nfsdata/environ/arno/figures/
figure(1); print -deps compare_hg;
figure(2); print -deps compare_wt;
cd ~/matlab/Arno
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

References

Beven, K., Kinematic subsurface stormflow, *Wat. Resour. Res.* 17(5), 1419–1424, 1981.

Bixio, A. C., S. Orlandini, C. Paniconi and M. Putti, Physically-based distributed model for coupled surface runoff and subsurface flow simulation at the catchment scale. In: Bentley, L. R. (ed.) *Computational Methods in Water Resources, Vol. 2.* Balkema, Rotterdam, The Netherlands, pp 1115–1122, 2000.

Boussinesq, J., Essai sur la theorie des eaux courantes, *Mem. Acad. Sci. Inst. Fra.* 23(1), 252–260, 1877.

Brooks, R. H. and A. T. Corey, Hydraulic properties of porous media, *Hydrology Paper* 3, 1964. Colorado State University, Fort Collins, CO.

Brutsaert, W., The unit response of groundwater outflow from a hillslope, *Wat. Resour. Res.* 30(10), 2759–2763, 1994.

Childs, E. C., Drainage of groundwater resting on a sloping bed, *Water Resour. Res.* 7(5), 1256–1263, 1971.

Fan, Y. and R. L. Bras, Analytical solutions to hillslope subsurface storm flow and saturation overland flow, *Water Resour. Res.* 34(4), 921–927, 1998.

Huyakorn, P. S., S. D. Thomas and B. M. Thompson, Techniques for making finite elements competetive in modelingflow in variably saturated porous media, *Water Resour. Res.* 20(8), 1099–1115, 1984.

Orlandini, S. and R. Rosso, Diffusion wave modeling of distributed catchment dynamics, *J. Hydrol. Engrg., ASCE* , 1996.

Paniconi, C. and E. F. Wood, A detailed model for simulation of catchment scale subsurface hydrologic processes, *Water Resour. Res.* 26(6), 1601–1620, 1993.

Paniconi, C. and M. Putti, A comparison of picard and newton iteration in the numerical solution of multidimensional variably saturated flow problems, *Water Resour. Res.* 30(12), 3357–3374, 1994.

Paniconi, C., A. G. J. Hilberts, E. E. van Loon and P. A. Troch, Preliminary description of a hillslope-storage Boussinesq model for subsurface and overland flow. Technical Report CRS4-TECH-REP-01/71, Center for Advanced Studies, Research and Development in Sardinia, Cagliari, Italy, 2001.

Paniconi, C., P. A. Troch, E. E. van Loon, B. Bijkerk and A. G. J. Hilberts, Behavior of a hillslope-storage Boussinesq model for subsurface stormflow and saturation overland flow, *Water Resour. Res.* , 2001 (in preparation).

Serrano, S. E., Analytical solutions of the nonlinear groundwater flow equation in unconfined aquifers and the effect of heterogeneity, *Water Resour. Res.* 31(11), 2733–2742, 1995.

Troch, P. A., C. Paniconi and E. E. van Loon, A hillslope-storage Boussinesq model for subsurface flow, *Water Resour. Res.* , 2001 (in preparation).

Troch, P. A., E. E. van Loon and A. G. J. Hilberts, Analytical solutions to a hillslope-storage kinematic wave equation for subsurface flow, *Adv. Water Resour.* , 2001 (submitted).

van Genuchten, M. T. and D. R. Nielsen, On describing and predicting the hydraulic properties of unsaturated soils, *Ann. Geophysics* , 1985.

Verhoest, N. E. C. and P. A. Troch, Some analytical solutions of the linearised Boussinesq equation with recharge for a sloping aquifer, *Water Resour. Res.* 36(3), 793–800, 2000.