Z-99 Geophysics with Parallel Spectral Elements

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Abstract

Spectral Elements have been applied to various applications in computational engineering, and enable reductions of computational resources respect to more traditional techniques like finite elements or finite differences. The main advantage is their high order approach, which equals the accuracy of low order methods while showing computational efficiency and suitability to parallel computation. This work illustrates how Spectral Elements can be used to solve various problems usually encountered in geophysics, such as the scalar acoustic equation and linear and non-linear elasticity.

Introduction

Is there any place in geophysics to excel and overcome current finite difference (FD) tools? Wave propagation, analysis of failures and fractures, solid-fluid interactions, anisotropy or scattering are examples of partial differential equations (PDE) traditionally faced in geophysics. Reasons for using FD are basically two: easy implementation and speed. However, high order numerical methods have recently gained popularity amongst mathematicians and engineers. Reasons are: better accuracy, flexibility and robustness. We have implemented a set of computational tools based on SE and aimed to solve problems frequently encountered in oil E&P: acoustic wave propagation, elastodynamics, Helmholtz equation for scattering, non-linear elasticity, etc. These integrated solvers exploit the computational power of multiprocessor systems through unstructured domain decomposition approaches. In this work we have applied our solvers to the solution of complex and challenging problems related to elasticity and acoustics. While efficiency and speed is still the door to further developments, there is an increasingly wider range of joint problems that can be solved accurately, thus incorporating more physics into our simulations.

Spectral Element Advantages

We refer the reader to Quarteroni and Valli (1994) if interested in the mathematical formulation of the various problems we solve in this work, since Spectral Elements is based on the variational formulation of the involved PDE's. For the sake of simplicity and the lack of space, we will refer in this paper only to concepts and implementation issues rather than mathematics.

Spectral elements can be regarded as an h-p extension of the common h method based on Finite Elements, based on the use of high order piecewise polynomial functions (Komatitsch and Vilotte, 1998). The great asset of the method is the capability to provide an arbitrary increase of accuracy simply enhancing the algebraic degree of these functions, the so-called spectral degree, accomplished by choosing Legendre-Gauss-Lobatto nodes for evaluating of

the integrals that appear in the weak formulation. When the "exact" solution u is regular, the numerical solution $u_{h,p}$ is expected to converge to u as

$$\left\| u - u_{h,p} \right\|_1 \leq C\, h^p\, e^{-p}$$

In practice, changing the order of approximation can be done transparently without changing the characteristics of the initial problem (fig. 1). Obviously, increasing the spectral degree p has also the effect to rise up the computational complexity. On the other hand, one can also play on the grid refinement h to improve the accuracy of the numerical solution (fig. 2). Both steady and time-dependent problems produce an algebraic system that can be solved (at each time step in the latest case) using parallel iterative procedures based on ghost points for efficient computation and minimizing communication amongst processors.

Numerical Examples

In this section we provide a couple of experiments aimed to show accuracy of the method. The first one is an example of the solution of the classical acoustic wave equation, so crucial in E&P geophysics. Finally we include a synthetic case of non-linear elasticity to give evidence of flexibility of our spectral element approach.

The Acoustic Wave Equation

First, we solve the acoustic wave equation on a regular grid for the analytical solution $u(x,y,t) = \sin(4\pi x)\cos(4\pi y)\cos(2\pi t)$ and measure the error given by $\varepsilon = \left\| u - u_{h,p} \right\|_0 / \left\| u \right\|_0$ for different values of p and h. Figure 4 shows the measured error in logarithmic scale for fixed spectral degree p and also for fixed element size h.

Figure 5 correspond to a salt-dome geometry proposed by Dablain (1986) with heterogeneous properties. The model dimensions are 10 km x 3 km. A fully unstructured mesh of approximately 4x104 quadrilaterals and spectral degree 4 was used, corresponding to more than 6x105 grid points. The picture reports various snapshots obtained with a point source of 50 Hz buried at 100 m depth over the dome.

The Elasticity Problem

Using SEM it is possible to quantify the influence of individual fractures and fracture populations on transport properties of joints. By coupling Griffith theory to stress wave propagation the geoscientist can reproduce numerical experiments for a better understanding of the evolution of cracked soils and occurrence of T-Y junctions in elastic media. Figure 6 shows the evolution of a cracked 2D medium under tensile stresses.

Conclusions

We have presented a solver for the solution of steady and time-dependent problems, based on spectral elements and domain-decomposition for parallel computing. A major asset of the technique is the flexibility in addressing complex geometries and large-scale problems. Numerical experiments have shown that the method is appropriate for linear and non-linear problems. Examples from acoustic wave propagation and non-linear crack population enforce us to integrate progressively more physical phenomena in our simulations.

Acknowledgements

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References

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- Komatitsch, D. and J.P. Vilotte, *The spectral element method: an efficient tool to simulate the seismic response of 2D and 3D geological structures*, BSSA, **88**, 368-392, 1998.

Figures

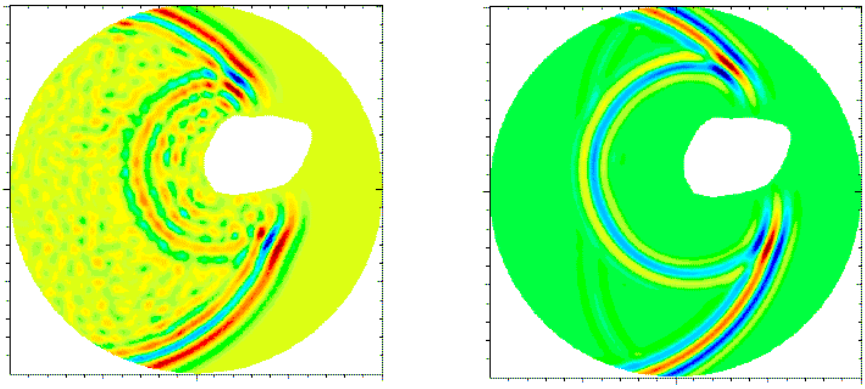

Figure 1. On the left, SE acoustic wave simulation with *p=1*. Numerical dispersion implies insufficient accuracy of the method. The option in FD/FE is to refine the grid. On the right, the order has been increased to *p=2*. Accuracy in the solution is finally obtained.

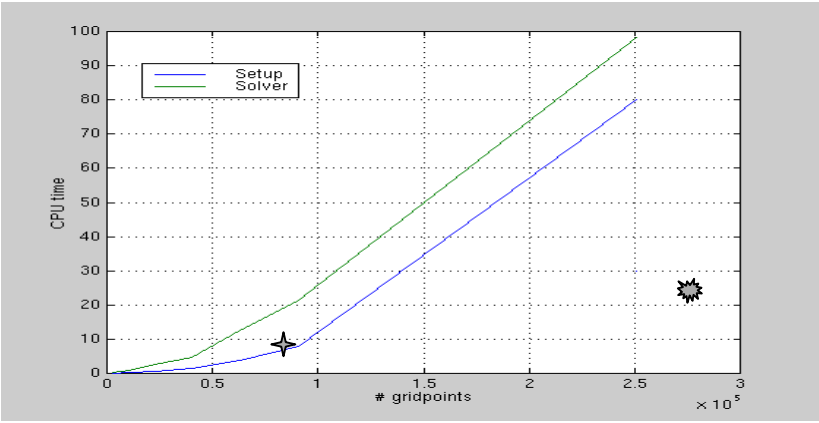

Figure 2. CPU time using SE with different grid sizes. The 8-point star shows the timing for an explicit 4-th order FD simulation of an acoustic wave with maximum grid points. The 4-point star corresponds to the time used by SE using higher order and same accuracy as the FD result, but less grid points.

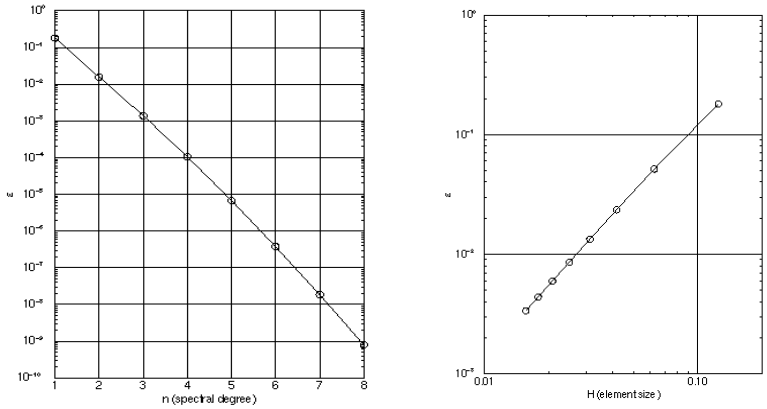

Figure 4. Accuracy of the numerical solution with respect to spectral degree *p* (left) and element size *h* (right).

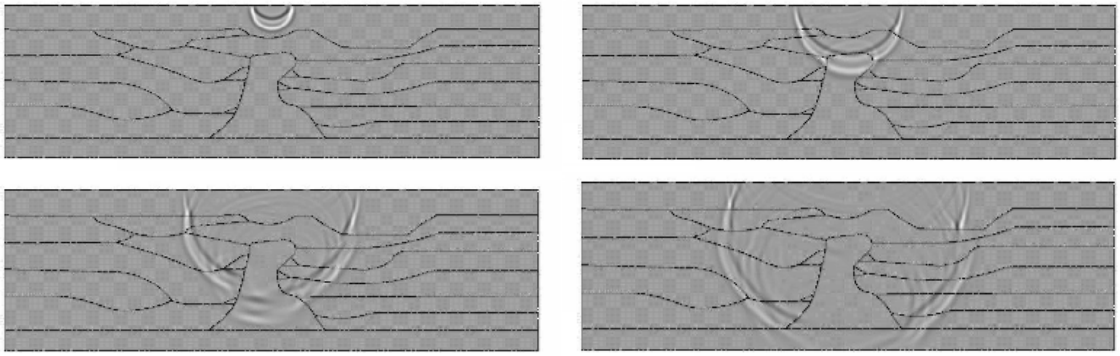

Figure 5. Snapshots of the salt-dome at T=0.25 s, T=0.5 s, T=0.75 s and T=1.0 s.

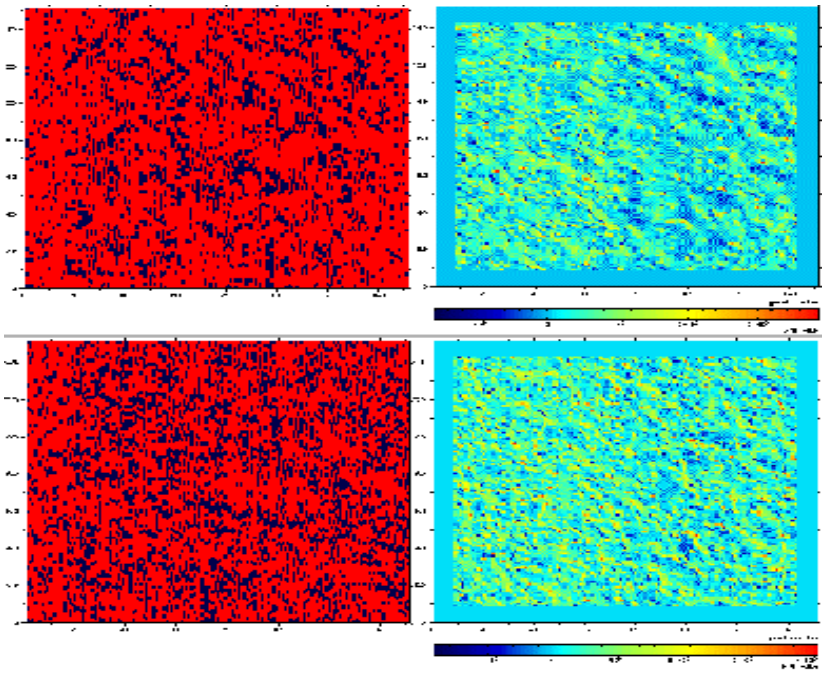

Figure 6. Crack population for a medium subject to tensile stresses. On the right, the normal stress field is displayed after a simulation based on SE with Griffith criterion for non-linear behavior of the fractures.