A Velocity Model Inference from the Inversion of 3D Zero-Offset Seismic Traces in the Space-Frequency Domain

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Structure of the Presentation

- Problem Formulation
- Non-Linear Inversion Framework
- Gradient Based Optimization
- 1D Numerical Examples
- Future Plans
- Conclusions

Problem formulation

$\bowtie$ problem context

- non-invasive subsoil imaging for hydrocarbon prospecting

$\bowtie$ problem statement

- reconstruction of scalar velocity field (v) of the subsoil
- given: . field measurements of acoustic traces, and
  . a wave progation code
- find v such that $\| S^* - S(v) \|$ is minimized

$S(v)$ → simulated data

S^* → measured data

Problem formulation

$\bowtie$ data

industrial context: inversion of large data sets (500 MB - 1GB)

data compression . coincident source-receivers (post-stack data)

. temporal frequency domain:

$FFT\{S(t), t \rightarrow \omega\} = \widehat{S}(\omega)$

$\Rightarrow$ inversion of 3D data sets feasable

$\bowtie$ wave propagation

one-way wave equation in frequency domain

solved by phase shifting

$\Rightarrow$ highly innovative approach $\Leftarrow$

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  - Wave Extrapolation
  - Direct Field Equation
  - Adjoint Field Equation
  - Gradient
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Wave Extrapolation

- $\bowtie$ p acoustic wave field: $\triangle_{(x,z)}\, p = \frac{1}{v^2}\, p_{tt}$
- $\bowtie$ $\widehat{p}(k_x, z, \omega) = FFT[p(x,z,t), x \rightarrow k_x, t \rightarrow \omega]$
- $\bowtie$ no lateral velocity variations: $v = v(z)$

  $\widehat{p}(z \pm \triangle\, z) = \exp(\pm j\, k_z\, \triangle\, z)\, \widehat{p}(z)$ $\qquad k_z = [(\omega/v)^2) - k_x^2]^{1/2}$

  (+)downward/(-) upward propagation

  $\Rightarrow$ phase-shift (PS) algorithm (Gazdag '78)

- $\bowtie$ with lateral velocity variations: $v = v(x,z)$

  intepolation procedure on $v = v(x,z)$ at each depth

  $\Rightarrow$ phase-shift plus interpolation (PSPI) algorithm (Gadzag-Sguazzero '83)

Direct Field Equation

$\bowtie$ Upward wavefield extrapolation - Demigration

$P_N = q_N$	N # layers in depth
$P_n = G(v_{n+1})\, P_{n+1} + q_n$	$n = N-1, \ldots, 1$
with $G(v_{n+1})$ the propagation filter	

$\bowtie$ P_1 simulated surface data ($S(v) = P_1$)

$\bowtie$ reflectivity q_n
- $*$ isosurfaces of discontinuity of the velocity
- $*$ computed by edge-detection filter

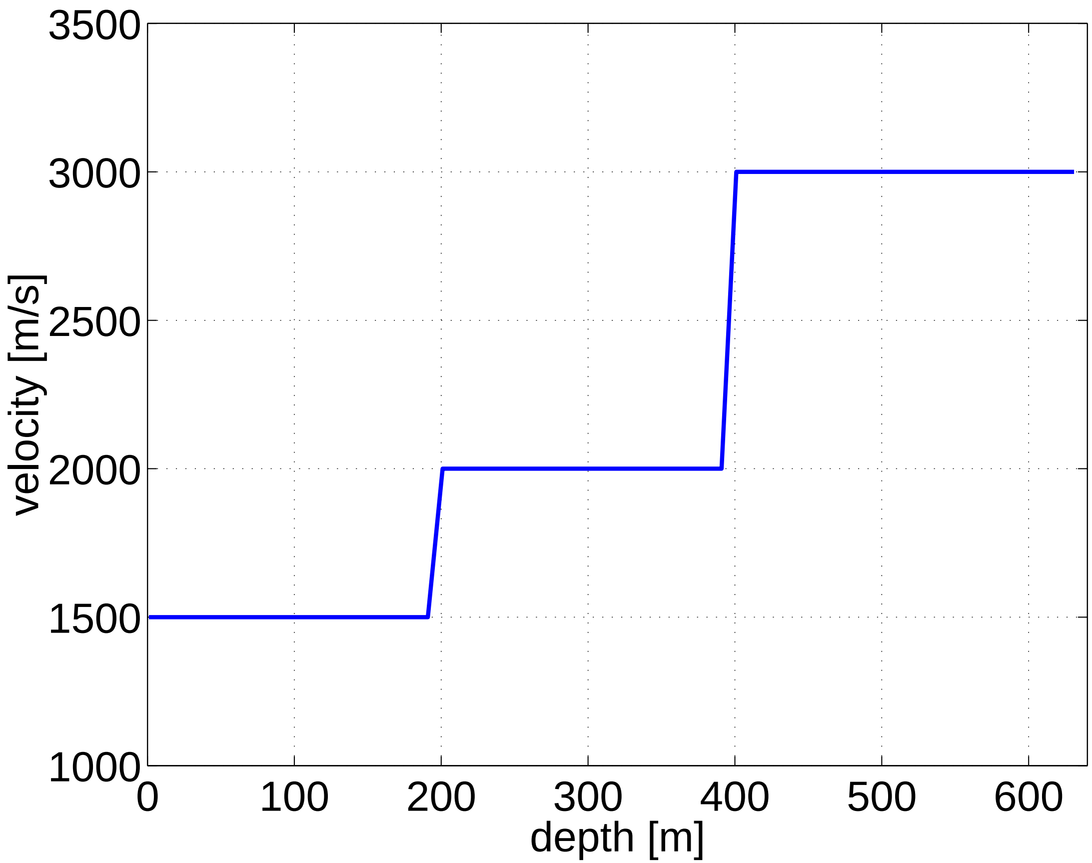
$\bowtie$ no lateral velocity variations

$$G(v_{n+1}) = \exp[-j\, k_z(v_{n+1})\ \triangle\ z] \quad \text{and} \quad q_n = \frac{v_{n+1} - v_n}{v_{n+1} + v_n}$$

Direct Field Equation

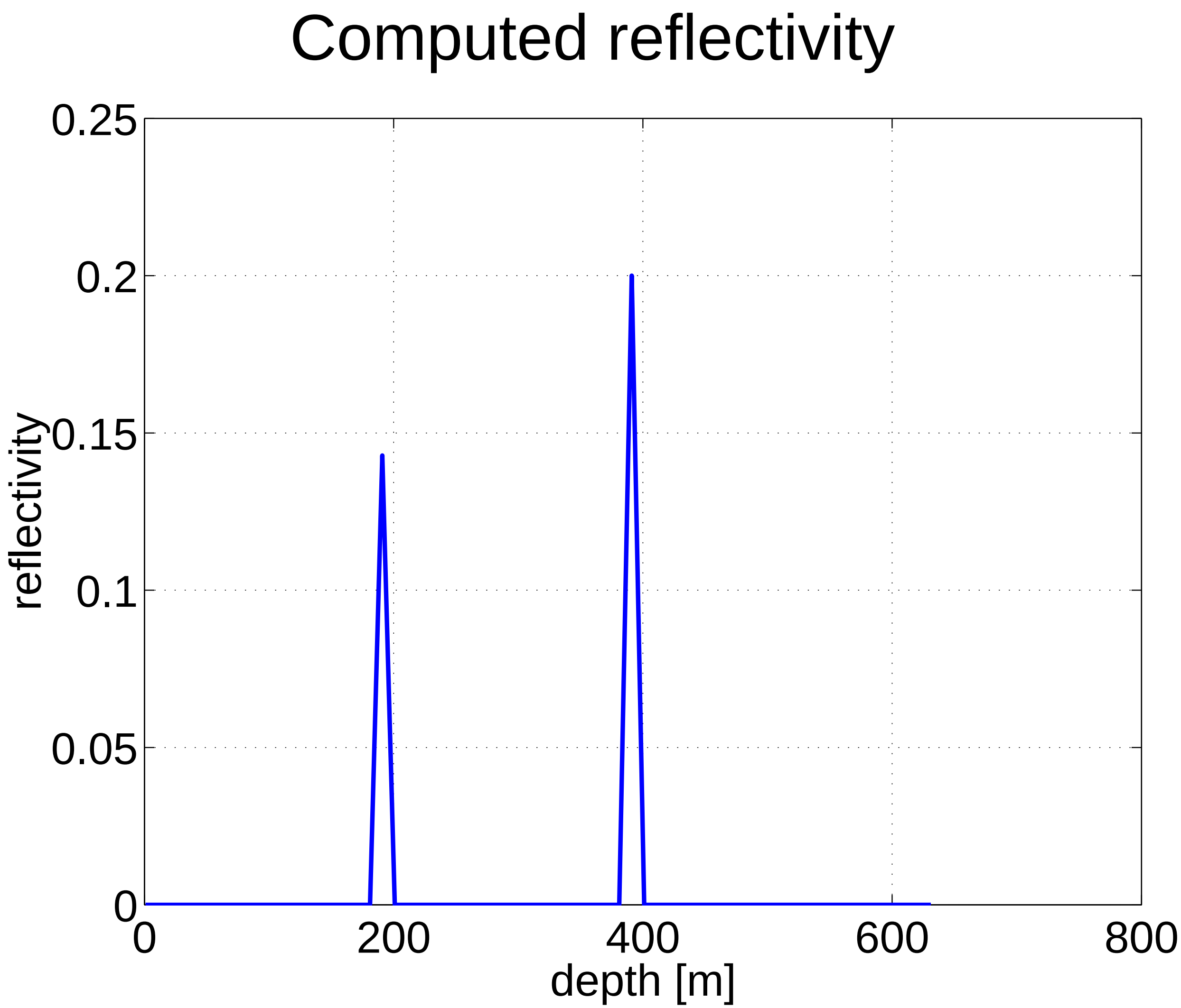
Example in 1D

Given velocity as of function of depth



$$N(\#\text{layers}) = 64$$
$$dz = 10m$$
$$z_{max} = 640m$$

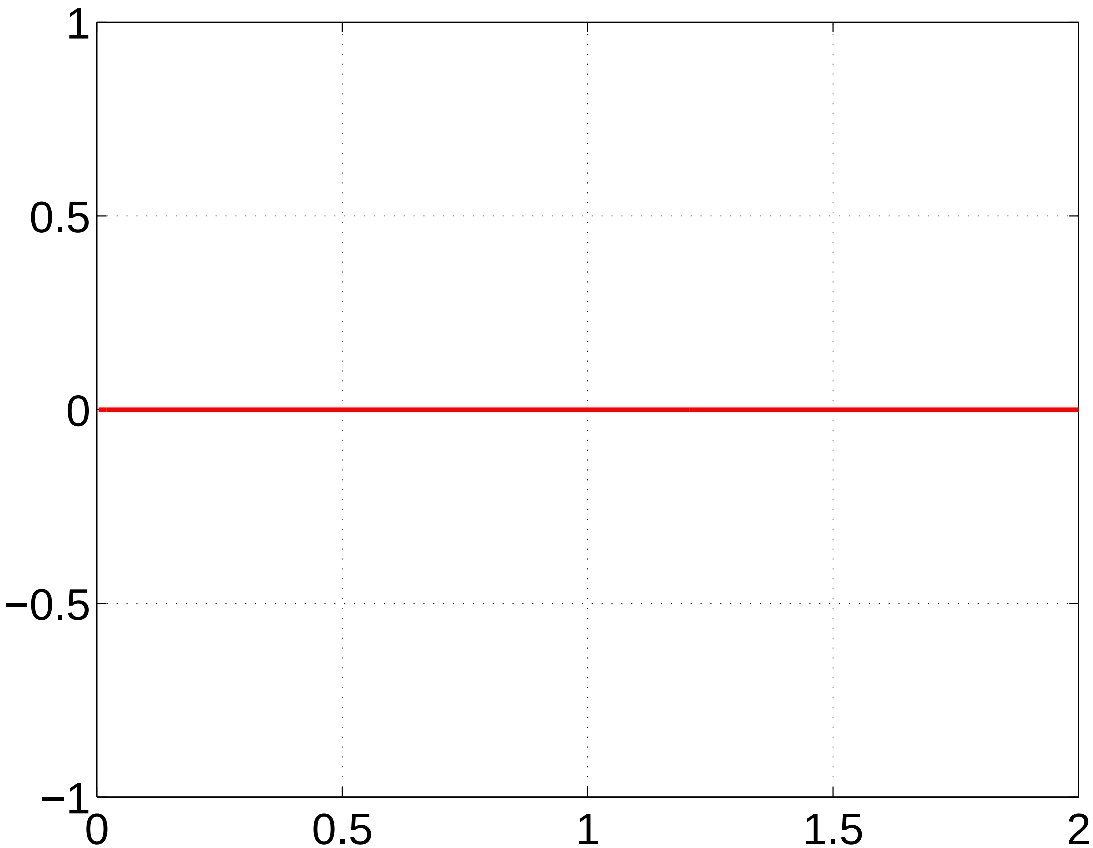
Direct Field Equation



Direct Field Equation

Acoustic field P_N at $z = 640m$

Initial condition (time domain)

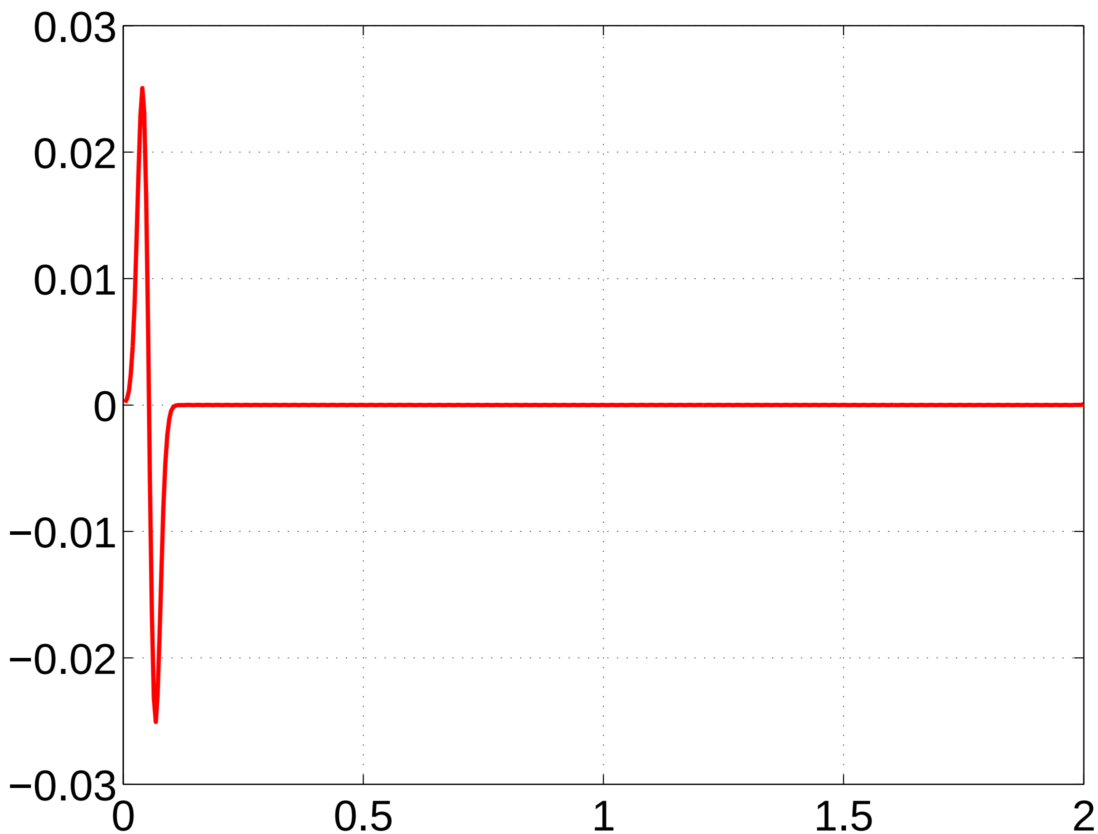


$n = 64$

Direct Field Equation

Acoustic field P_n at $z = 350m$

Hit first velocity discontinuity

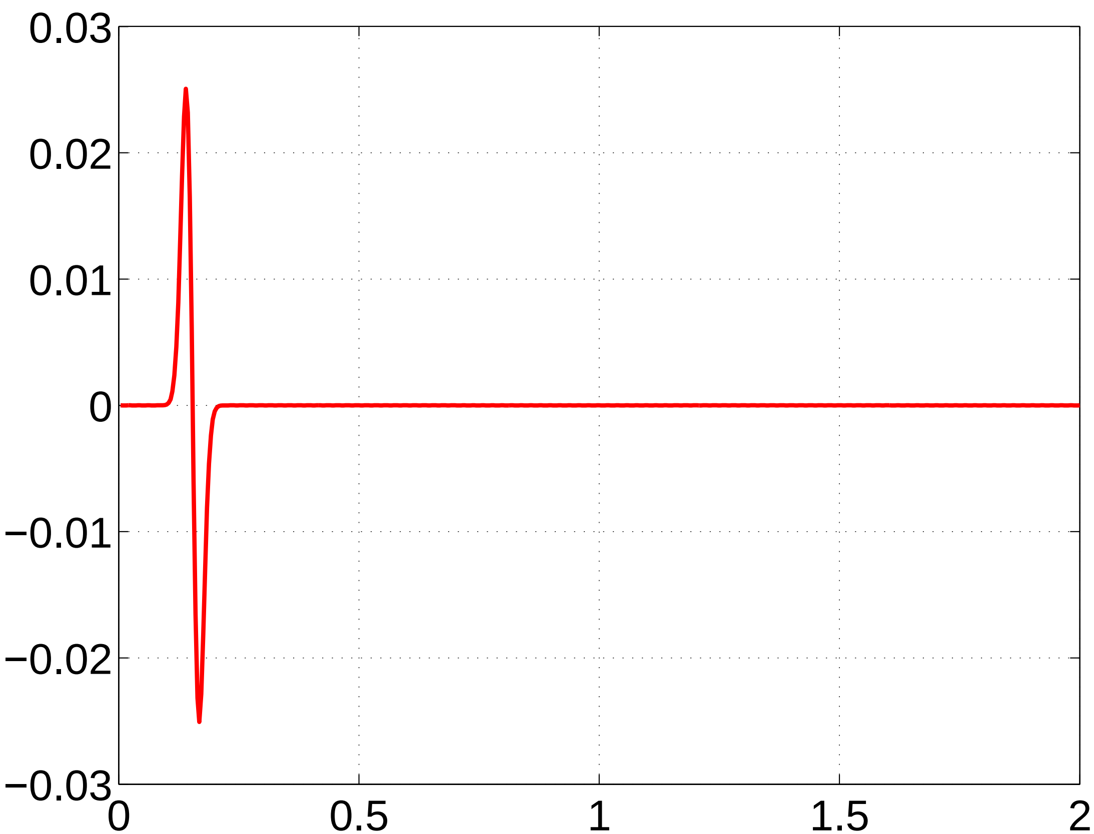


$n = 35$

Direct Field Equation

Acoustic field P_n at $z = 250m$

First discontinuity further propagated

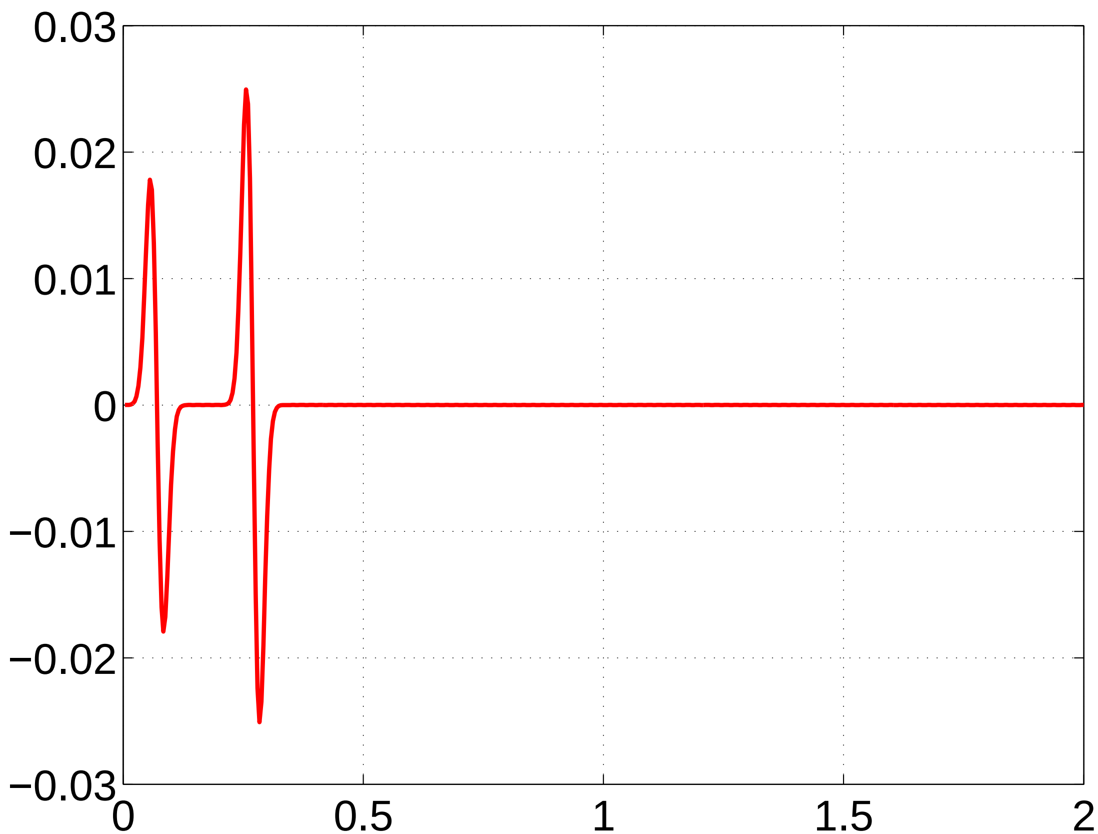


$n = 25$

Direct Field Equation

Acoustic field P_n at $z = 150m$

Hit second velocity discontinuity

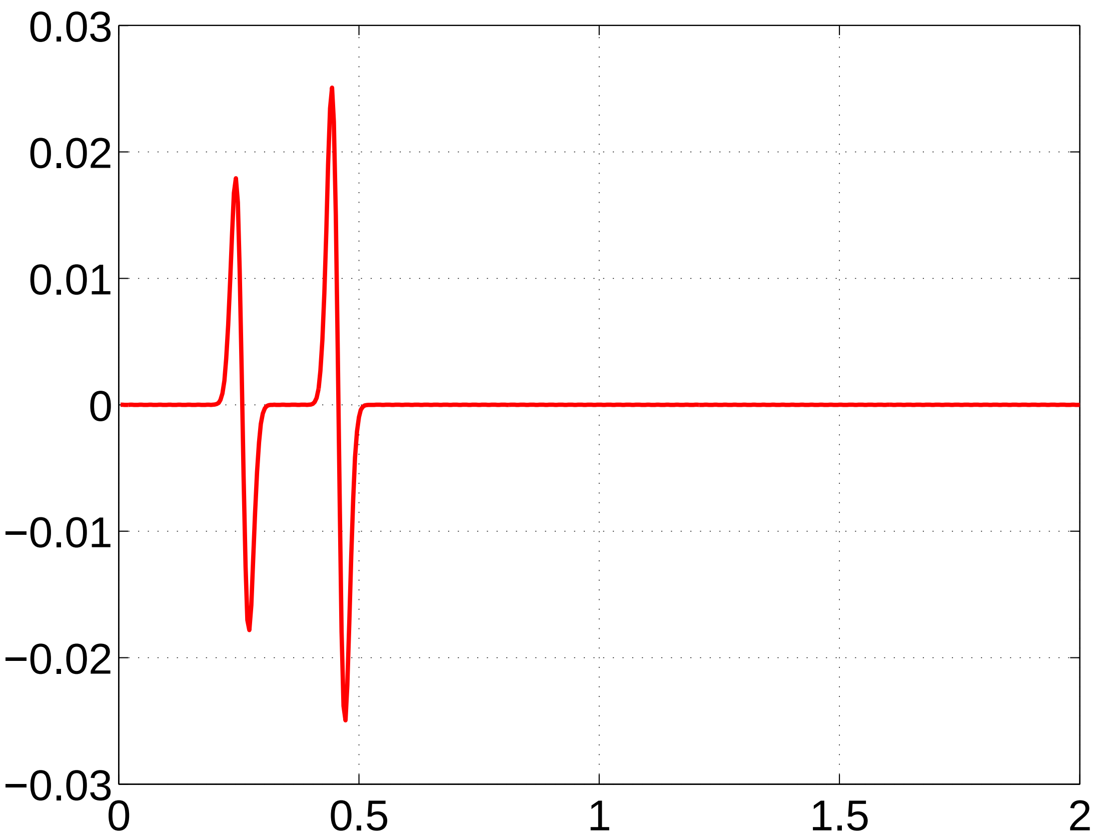


$n = 15$

Direct Field Equation

Acoustic field at the surface $(z = 0)$

Simulated data $S(v)$



$n = 1$

Adjoint Field Equation

- $\bowtie$ $\mathcal{L}(v) = \| S^* - S(v) \| + \sum_n \langle \lambda_n, (P_{n+1} - G(v_{n+1}) P_n - q_n) \rangle$

  Lagrange multiplier

- $\bowtie$ Downward wave extrapolation
  Migration for λ_n

$$\begin{array}{lcl} \lambda_1 & = & S^* - S(v) \\ \lambda_{n+1} & = & G^+(v_{n+1})\, \lambda_n \end{array}$$

- $\bowtie$ no lateral velocity variations
  $G^+(v_{n+1}) = G^*(v_{n+1}) = \exp[j\, k_z(v_{n+1})\ \triangle\ z]$

- $\bowtie$ with lateral velocity variations
  using interpolation (cfr. demigration)

Gradient

- $\frac{\partial \mathcal{L}}{\partial v} = \sum_{n=1}^{N-1} \langle \lambda_n, \frac{\partial G}{\partial v}(v_{n+1})\, P_{n+1} \rangle + \sum_{n=1}^{N} \left\langle \lambda_n, \frac{\partial q_n}{\partial v} \right\rangle$
- direct field P_n : $z \rightarrow z + \triangle z$
  adjoint field λ_n : $z \rightarrow z - \triangle z$
  $\Rightarrow$ synchronization required
- no lateral velocity variations
  $\frac{\partial G}{\partial v}(v_{n+1})$ and $\frac{\partial q_n}{\partial v}$ can be computed analytically
- with lateral velocity variations
  $\frac{\partial G}{\partial v}(v_{n+1})$ requires derivative of the PSPI interpolation operator

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Gradient based optimization

- conjugate gradient (CG) method for optimization
  1. determine search direction
  2. perform line-search along established direction
- projected conjugate gradient (PCG) method
  $v_{\text{min}} \leq v \leq v_{\text{max}}$
  project iterate onto set of feasable solutions
- local optimization method
  susceptible to local minina
  enhancements of PCG required

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1D Numerical examples

Test case scenario

1. Generate measured stack (surface data)

   exact velocity + computed reflectivity $\overset{\text{DEMIG}}{\Rightarrow}$ measured surface stack : $S(x, y, \omega)$

2. Generate simulated stack
   perturb velocity

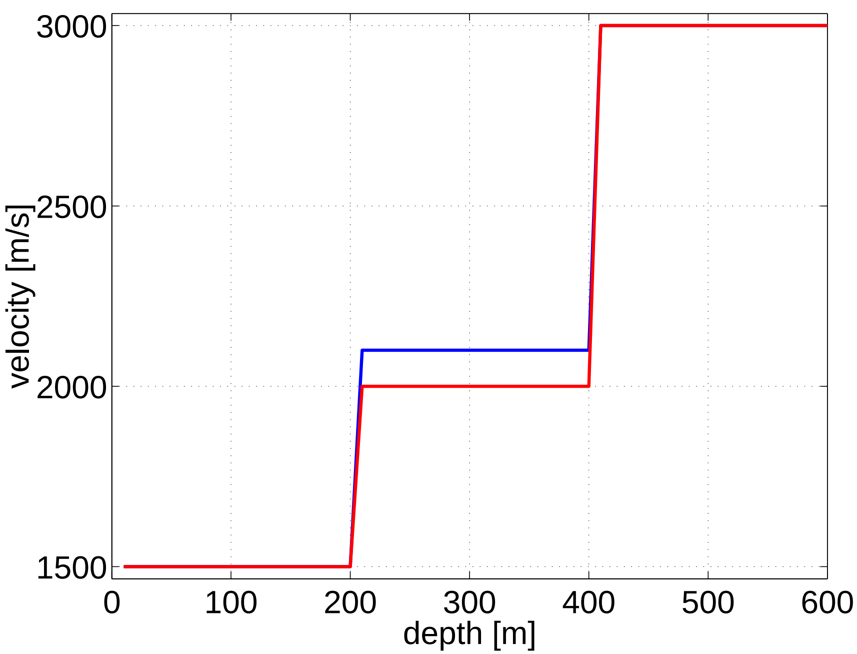
   initial guess vel. + comp. reflec. $\overset{\text{DEMIG}}{\Rightarrow}$ simulated surface stack : $P(x, y, \omega)$

3. Reconstruct velocity
   iterative updating of the velocity (v) by PCG algorithm
   search v such that $\| S - P(v) \|$ is minimized

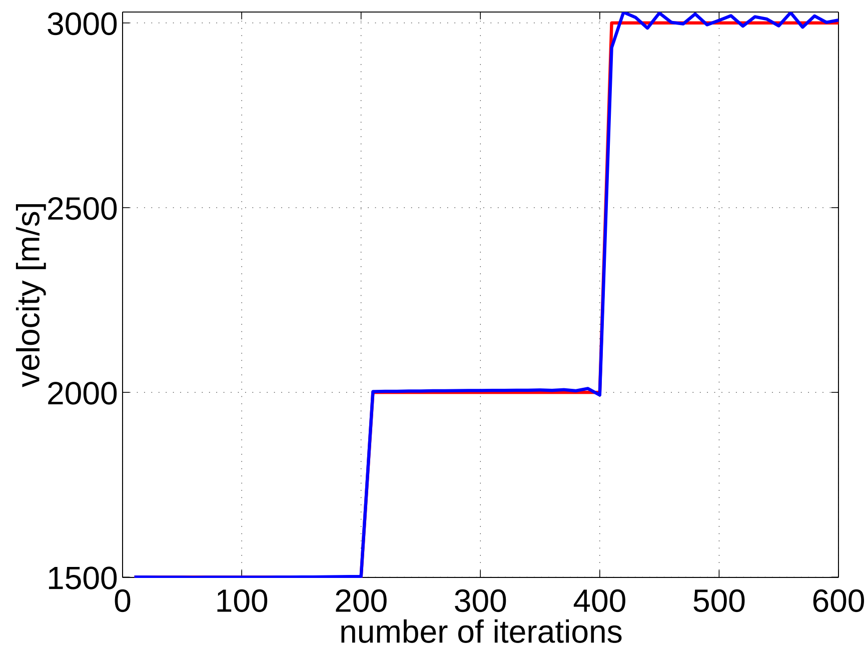
First Numerical example

Approximate and exact solution

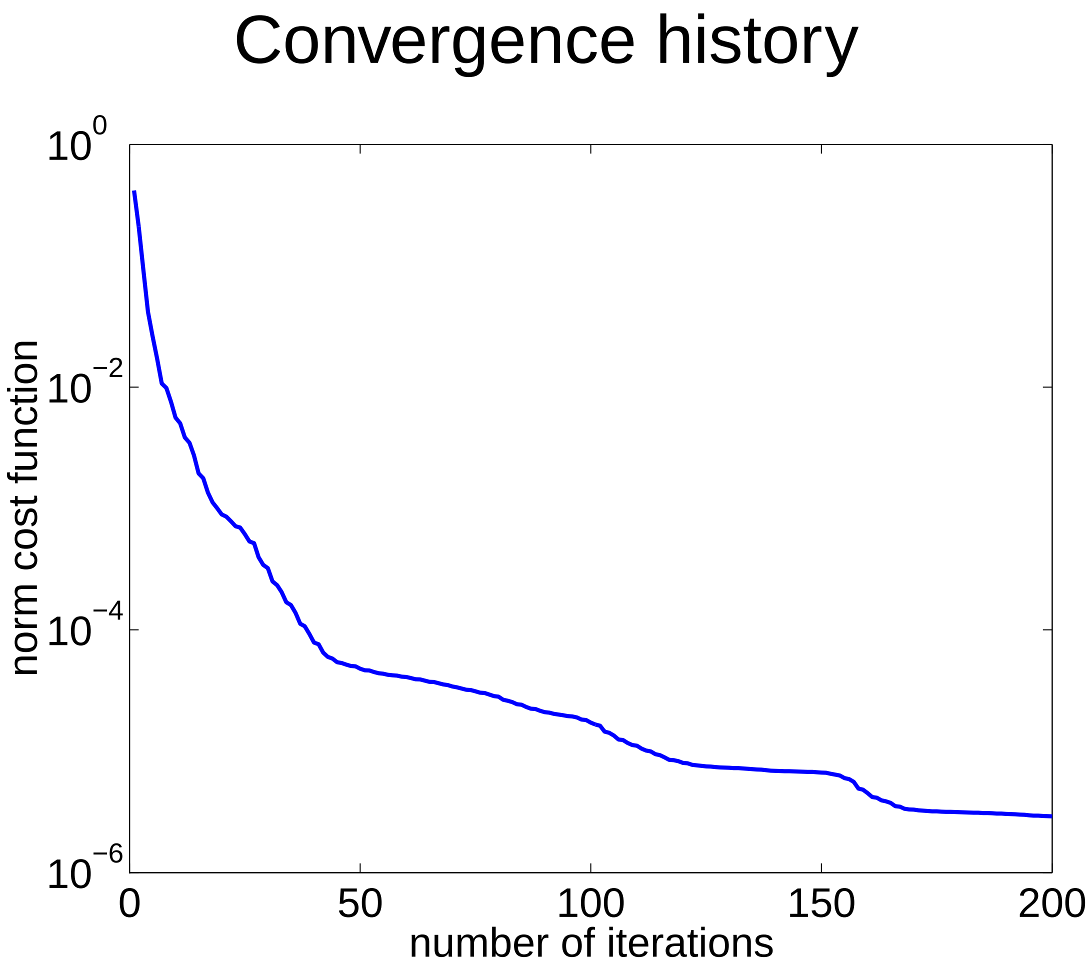
start solution



final solution (200 PCG it.)



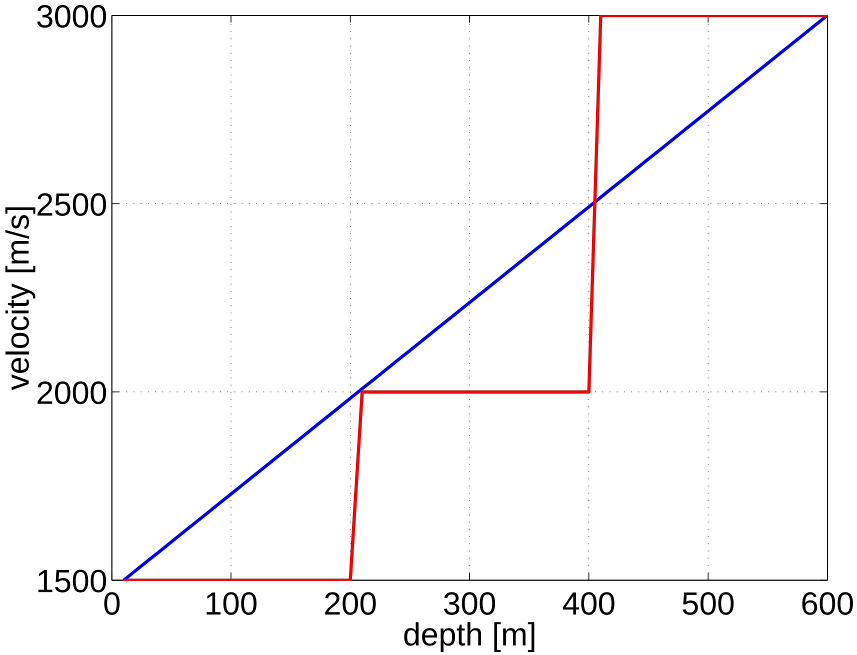
First Numerical example



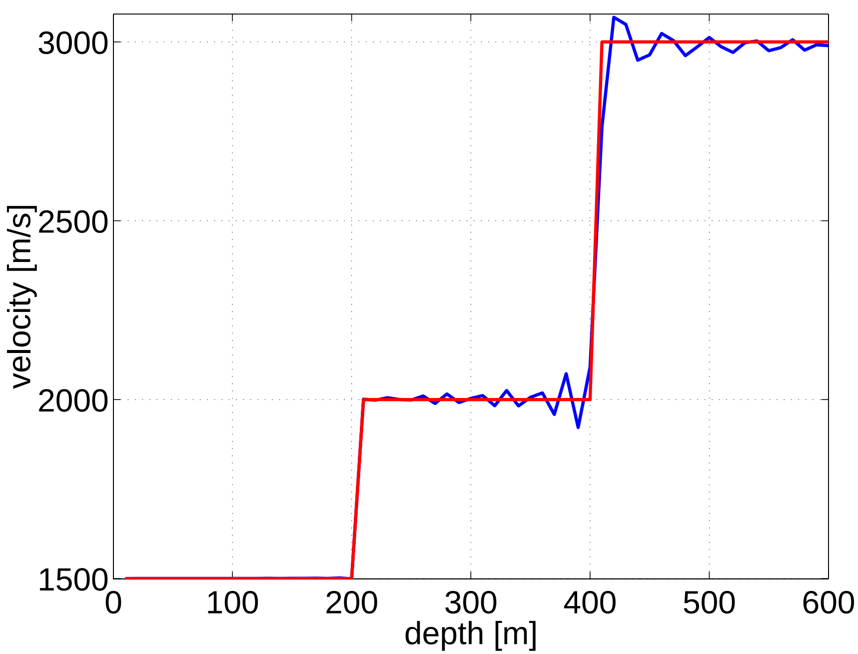
Second Numerical example

Approximate and exact solution

start solution



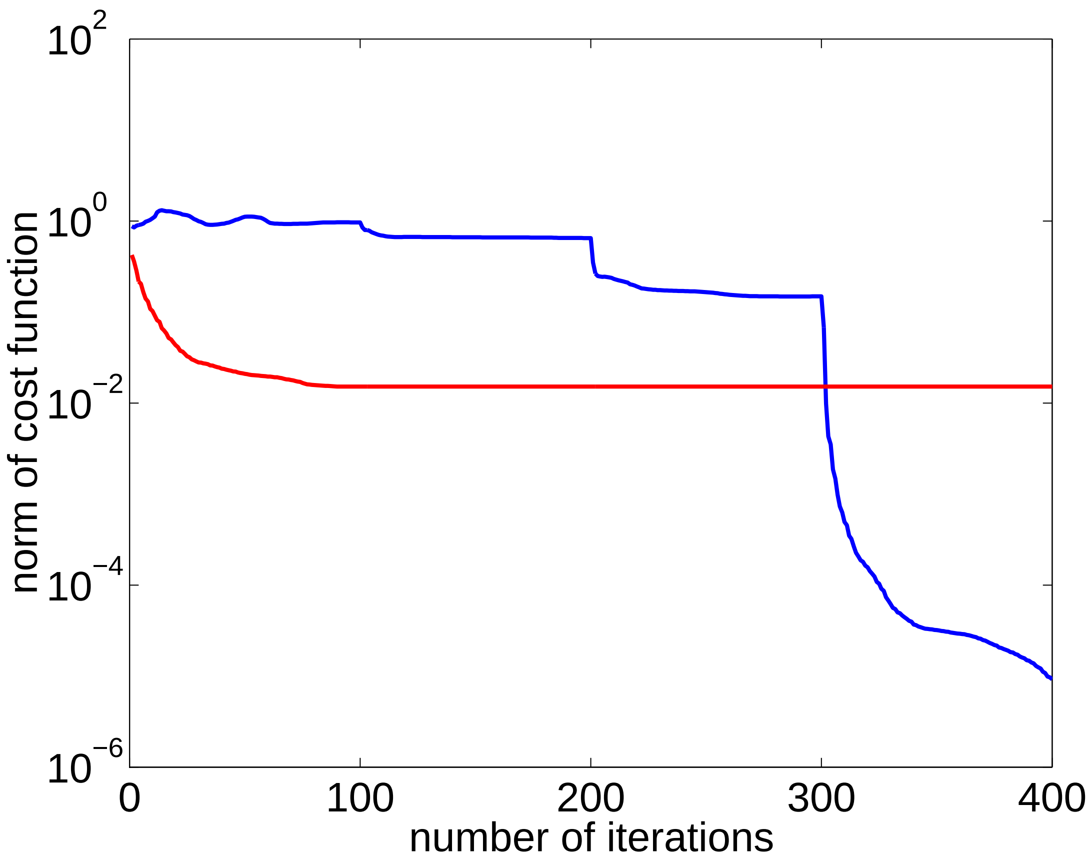
final solution (400 PCG it.)



Second Numerical example

Convergence history

one-level approach - four-level approach



Future Work

- $\bowtie$ Further algorithmic developments in 1D
  - $*$ Accelerate speed of convergence
  - $*$ Avoid local minina
  - $*$ Automate multilevel approaches

- $\bowtie$ Extension to 2D and 3D
  - $*$ Computation of $\frac{\partial G}{\partial v}$ in case of PSPI
  - $*$ Computation of reflectivity
  - $*$ Alignment of direct and adjoint fields in gradient computation
  - $*$ ...

Conclusions

- We proposed a new algorithm for the reconstruction of the scalar velocity field of a subsoil from post-stack seismic data.
- The algorithm is based on a non-linear Lagrangian framework in which direct and adjoint equations are solved by demigrating and migrating in the frequency domain respectively.
- We presented encouraging preliminary results for 1D synthetic test cases.
- Future work is required to extend this work in 2D and 3D.