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Turbulent Prandtl Number, reformulation of Kay's correlation

Vincent Moreau, CRS4

1 Introduction

The primary coolant of some Gen-IV reactors under current development is a liquid metal thermally characterized by a low Prandtl number [1]. This leads to discrepancies in the modeling of the turbulent heat transfer in the boundary layer region if directly treated according to the Reynolds analogy. Many correlations have been derived to tackle the issue [2]. Almost all of them use global parameters such as the Reynolds or Peclet numbers which are not well defined in complex geometries and by such not suitable for the related CFD simulations. To the author knowledge, only Kay's correlation is using only local parameters. Several variants of the correlations have been used with significant success, leading to some perplexity. The scope of this paper is to show that the correlations can be simply derived on a basic assumption with regards to the non-linear combination of stochastic effects. The variants then come from different approximations of a mother formula.

2 Derivation of Pr_t and its approximations

In CFD codes, heat diffusion modelling and implementation is represent as the direct sum of two contributions, the molecular one and the turbulent one. Both contributions act in the same direction, proportionally to the local temperature gradient. Each contribution is also proportional to a conductivity coefficient. In other words the effective conductivity coefficient

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Corresponding Author: Vincent Moreau, moreau@crs4.it
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k_e is given as

$$k_e = k + k_t \tag{1}$$

where k is the molecular conductivity coefficient observed in static or laminar flows and k_t is the increment due to turbulence.

Conduction in general is originated from a stochastic process repeated a number of time. Molecular conduction and turbulent conduction look like different unrelated mechanisms operating at different scales. While molecular conduction is in fact a molecular process, turbulent conduction is rather based on a convective process. The cumulative effect is a convolution rather than a mere juxtaposition. In turn, one would expect the combined effects to be according to:

$$k_e = \sqrt{k^2 + k_0^2}, \tag{2}$$

where k_0 is the turbulent conductivity expected for a similar fluid with (almost) zero molecular conductivity or similarly when the turbulence is extremely large. We will now see what this simple hypothesis brings in term of the Prandtl number.

We introduce the thermal diffusivity $\alpha = \frac{k}{\rho C_p}$ with ρ the fluid density and C_p its specific heat. The (molecular) Prandlt numbers is defined as $Pr = \frac{\nu}{\alpha}$

where ν is the kinematic molecular viscosity. Similarly, the turbulent Prandtl number is defined as

$Pr_t = \frac{\nu_t}{\alpha_t}$

and the asymptotic turbulent Prandtl number as

$Pr_0 = \frac{\nu_t}{\alpha_0}$.

Noting that

$\frac{k}{k_0} = \frac{\alpha}{\alpha_0} = \frac{Pr_0 \nu}{Pr \nu_t}$,

elimination of k_e and simple algebraic manipulations from the first two equations give:

$$Pr_t = Pr_0(\sqrt{1 + (\frac{\alpha}{\alpha_0})^2} + \frac{\alpha}{\alpha_0}). \tag{3}$$

Expanding the square root term would give a second order term so at the first order we have:

$$Pr_t \simeq Pr_0(1 + \frac{\alpha}{\alpha_0}) = Pr_0(1 + \frac{Pr_0 \nu}{Pr \nu_t}). \tag{4}$$

Taking into consideration that Pr_0 is usually taken as $Pr_0 = 0.85$ and rewriting at first order this last equation in the usual form, we get:

$$Pr_t = 0.85 + \frac{0.7}{Pr \frac{\nu_t}{\nu}} \tag{5}$$

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which is exactly the form of the Kay correlation used in [3] [4] for application in CFD.

This first order approximation is however unsuited in case the $\frac{Pr_0\nu}{Pr\nu_t}$ is not everywhere small. Looking for an algebraic first order approximation of the square root term always valid regardless of the value of the ratio $\frac{\alpha}{\alpha_0}$, we find: $\sqrt{1+X^2} = (1+X)\sqrt{1-2\frac{X}{(1+X)^2}} \simeq 1 + X - \frac{X}{1+X}$ leading to

$$Pr_t = Pr_0(1 + 2\frac{Pr_0\nu}{Pr\nu_t} - \frac{\frac{Pr_0\nu}{Pr\nu_t}}{1+\frac{Pr_0\nu}{Pr\nu_t}}) \tag{6}$$

When $\frac{\alpha}{\alpha_0}$ is large, and keeping the constant part for compatibility, the approximation reduces to:

$$Pr_t = Pr_0(1 + 2\frac{Pr_0\nu}{Pr\nu_t}) = 0.85 + \frac{1.445}{Pr\frac{\nu_t}{\nu}} \tag{7}$$

While we can find a span of the second coefficient from 0.7 up to the value of 2 initially proposed by Kay, the current value of 1.445 is almost identical to the value value of 1.46 used by [5] to better fit heat transfer in a tube by direct analytical integration.

3 Conclusion

From an elementary consideration about independent stochastic process, we propose turbulent and molecular thermal conductivity to be square additive. As a direct consequence, comes an expression of the Prandtl number which first order approximations are identical to variants of the Kay correlation, the only non-trivial correlation exclusively based on local parameters and thus suited for arbitrary CFD simulations. An algebraic first order approximation giving the right asymptotic behaviors is also given. In case the Prandtl number formula brings improvement to the modeling, in particular to the modeling of the thermal boundary layer, it would be interesting to investigate whether the square additivity could be similarly extended to the viscosity and bring similar improvement to the viscous boundary layer modeling. This is however unfortunately far beyond the author's competency.

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